基于涡动力学的流体机械流动分析及优化设计

Flow Analyses and Optimization Design of Fluid Machines Based on Vortex Dynamics

清华大学能动系 祝宝山

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交流内容

- Vortical Flows in Fluid Machinery
- Lagrangian Vortex Method (LVM) and Applications
- Vortex Flow Control by Optional Design
- Summary

抽水蓄能机组

流体机械性能和运行特性关键：

①旋转能量利用

②旋涡流动控制

将涡动力学理论和流体机械流动理论相结合开展水力机械的内部流动分析和优化设计。

Vortex Generator and Utilizer

Basic relations for turbomachinery without considering viscosity:

Direct problem:

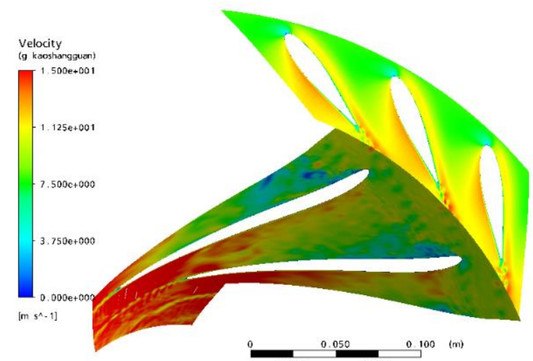
$$\frac{\partial E}{\partial l} = \varpi \frac{\partial \left(r v_\theta \right)}{\partial l}, \quad E = \frac{v^2}{2} + G + \frac{p}{\rho}$$

Inverse problem:

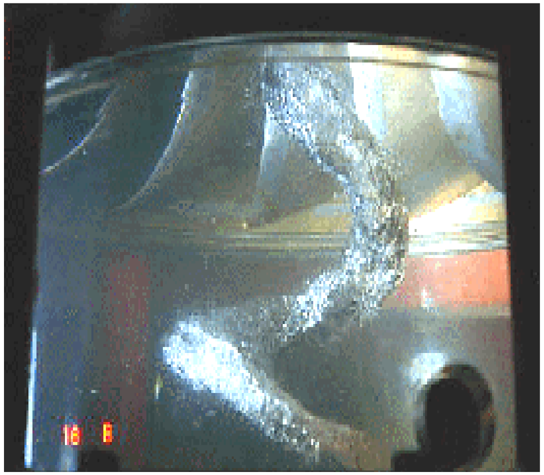
$$p^+ - p^- = \frac{2\pi}{Z} \rho W_{mbl} \frac{\partial \left(r v_\theta \right)}{\partial m}$$

Vortical Flows in Fluid Machinery

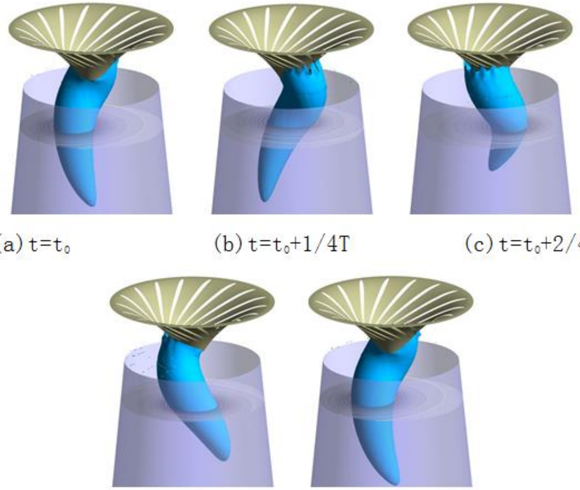
Besides prerotation, secondary flows, some vortical flows exist. Such as Karman vortices, Channel vortices…



Vortical Flows in Fluid Machinery

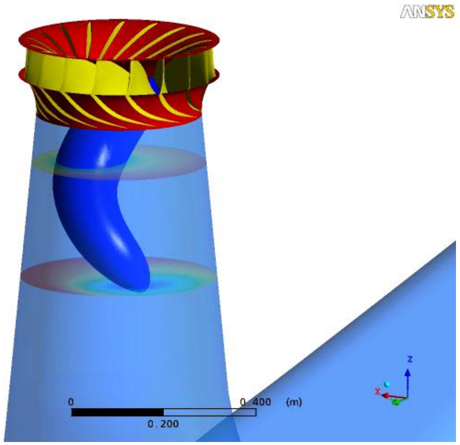


Vortex Rope

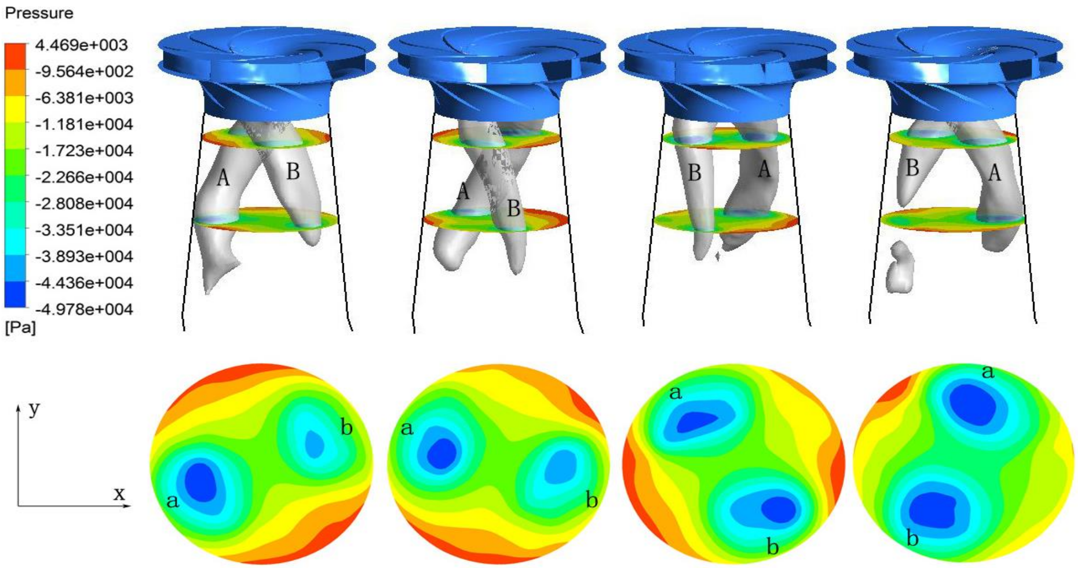


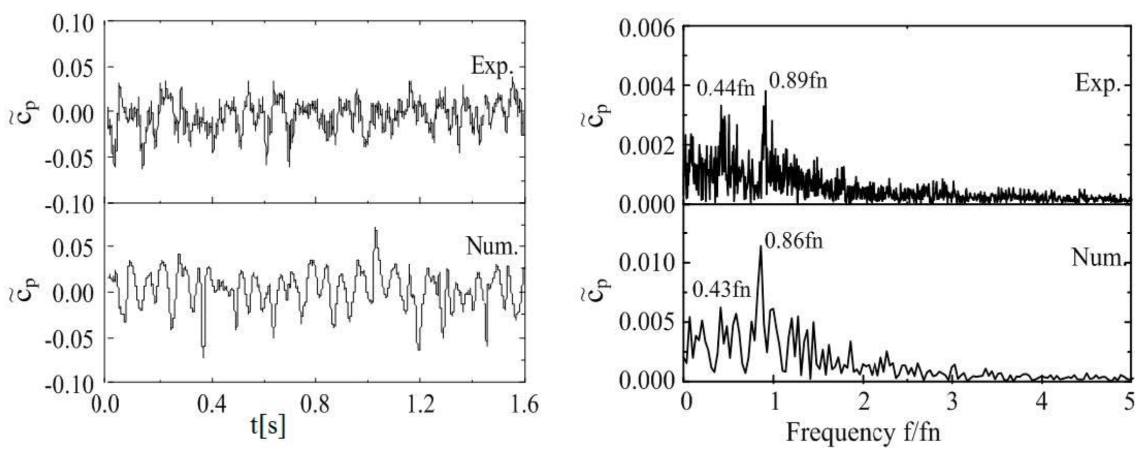
(a) $t=t_0$ (b) $t=t_0+1/4T$ (c) $t=t_0+2/4T$

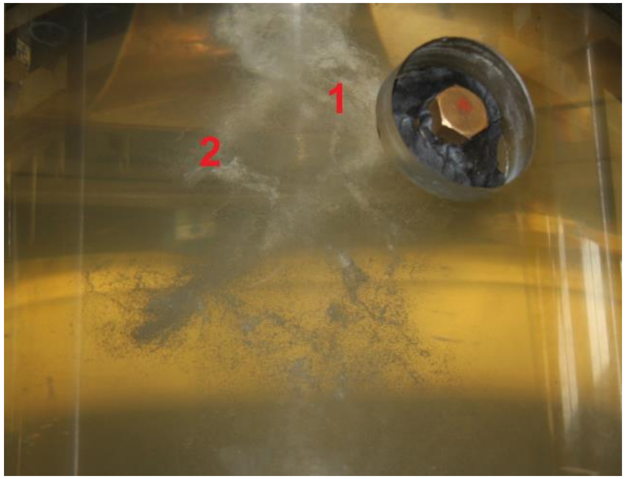
(d) $t=t_0+3/4T$ (e) $t=t_0+T$



Vortical Flows in Fluid Machinery







Twin-Vortex Rope

Lagrangian Vortex Method (VM)

N-S Eqns.

u-P Form:

$$\nabla \cdot \mathbf{u} = \mathbf{0}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\nabla p}{\rho} + \nu \Delta \mathbf{u}$$

u-ω Form:

$$\Delta \mathbf{u} = -\boldsymbol{\omega}$$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{u} \cdot \nabla)\boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{u} + \nu \Delta \boldsymbol{\omega}$$

Velocity : Integral Formulation

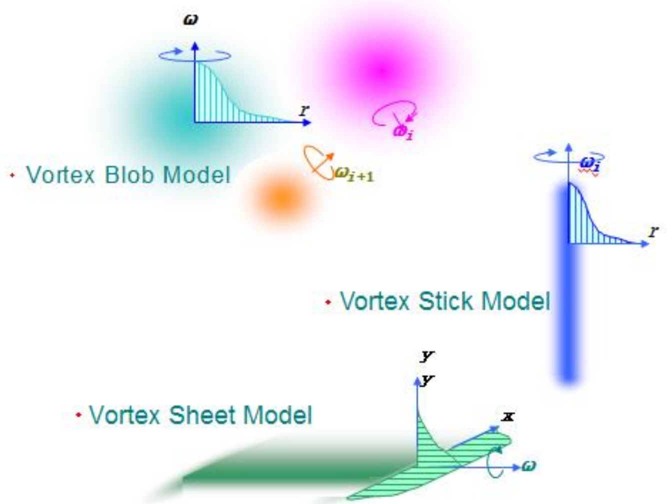
$$\mathbf{u}(\mathbf{r},t)=\int_V \boldsymbol{\omega}\times\nabla G dV+\int_S (\mathbf{n}\cdot\mathbf{u})\nabla G-(\mathbf{n}\times\mathbf{u})\times\nabla G dS$$

$$G=\begin{cases}\dfrac{1}{4\pi}\dfrac{\mathbf{r}}{|\mathbf{r}|^3} & \text{3D problem}\\[2ex] \dfrac{1}{2\pi}\dfrac{\mathbf{r}}{|\mathbf{r}|^2} & \text{2D problem}\end{cases}$$

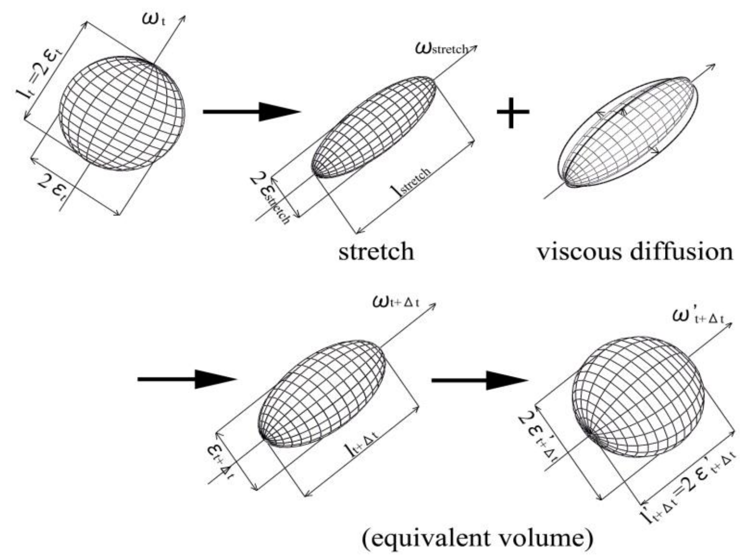
Vorticity: Lagrangian Scheme

$$\frac{d\mathbf{r}}{dt}=\mathbf{u},\qquad \frac{d\boldsymbol{\omega}}{dt}=\boldsymbol{\omega}\cdot\nabla\boldsymbol{u}+\nu\Delta\boldsymbol{\omega}$$

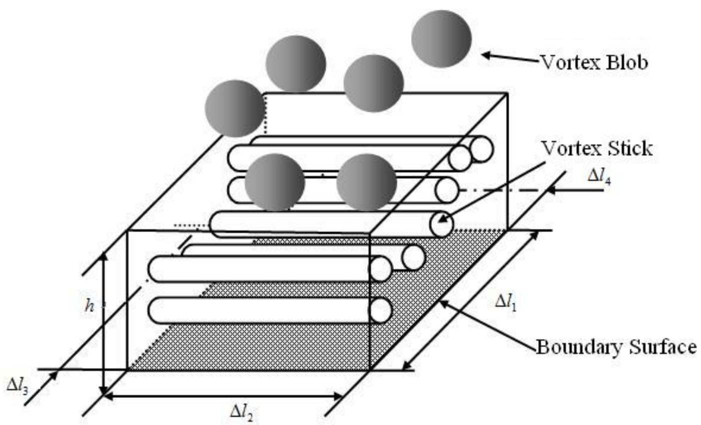
LVM and Applications



Discretization of Vorticity Field



Core Spreading Method



Three Dimensional Vorticity Boundary Condition

Viscous Disposal:

Particle Strength Exchange,
Core Spreading Method,
Random Walk Method, etc.

Pressure: Differential and Integral Methods

Differential Form:

$$\frac{1}{\rho}\mathbf{n}\times\nabla p=\nu\frac{\partial\boldsymbol{\omega}}{\partial\boldsymbol{n}}-\frac{1}{\rho}(\mathbf{n}\times\nabla)\cdot(\tau\mathbf{n})-\mathbf{n}\times\mathbf{a}_B$$

Integral Form:

$$\Delta H=-\nabla\cdot(\boldsymbol{u}\times\boldsymbol{\omega}),\ \ H=p/\rho+u^2/2$$

$$\beta H+\int_S H\left(\frac{\partial G}{\partial n}\right)dS=-\int_V\nabla G\cdot(\boldsymbol{u}\times\boldsymbol{\omega})\,dV-\int_S\left[G\cdot\boldsymbol{n}\cdot\left(\frac{\partial\boldsymbol{u}}{\partial t}\right)+\nu(\nabla G\times\boldsymbol{\omega})\cdot\boldsymbol{n}\right]dS$$

$$\beta=\begin{cases}1, & \text{Inside the flow field;}\\ 1/2, & \text{On a smooth boundary;}\\ 0, & \text{Outside the flow domain.}\end{cases}$$

Force and Moment: First Order Vorticity Moment;
Second Order Vorticity Moment.

$$F = -\frac{\rho}{2}\frac{D}{Dt}\left\{\int_V (\boldsymbol{r}\times\boldsymbol{\omega})\,\mathrm{d}V - \int_{\partial V}\boldsymbol{r}\times(\boldsymbol{n}\times\boldsymbol{V})\,\mathrm{d}S\right\}$$

$$M = \frac{\rho}{2}\frac{D}{Dt}\left\{\int_V r^2\boldsymbol{\omega}\,\mathrm{d}V + \int_{\partial V} r^2(\boldsymbol{n}\times\boldsymbol{V})\,\mathrm{d}S\right\}$$

Attractive Features

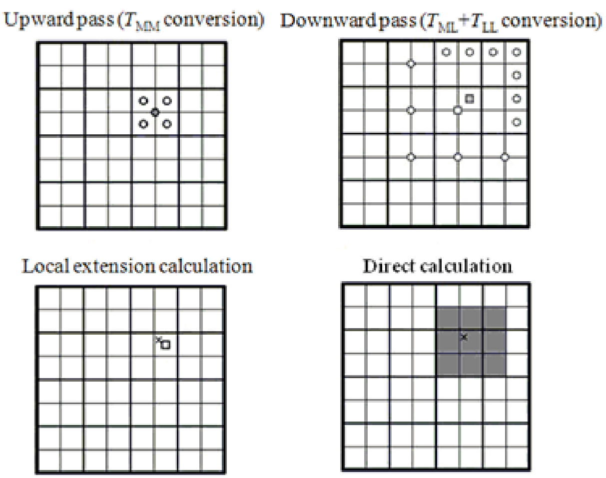
- Unnecessary of grid-generation;
- Easy to calculate moving or deforming boundary problem;
- Only need to calculate vorticity domain;
- Vorticity is calculated directly.

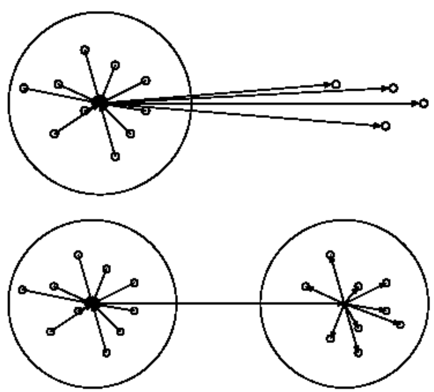
The Lagrangian vortex method is an ideal tool for numerical analysis of complex and vortical flows in fluid machinery.

Disadvantages and Improvements:

◆Reduce the integral calculation load

$$O(N^2) \rightarrow O(N)$$





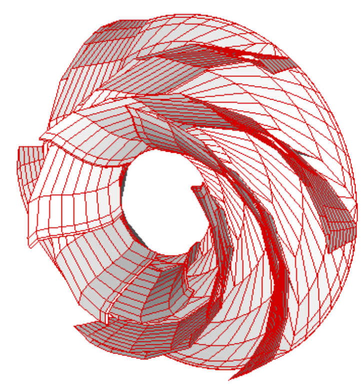
Fast Multipole Method

◆ Complex Solid Boundary

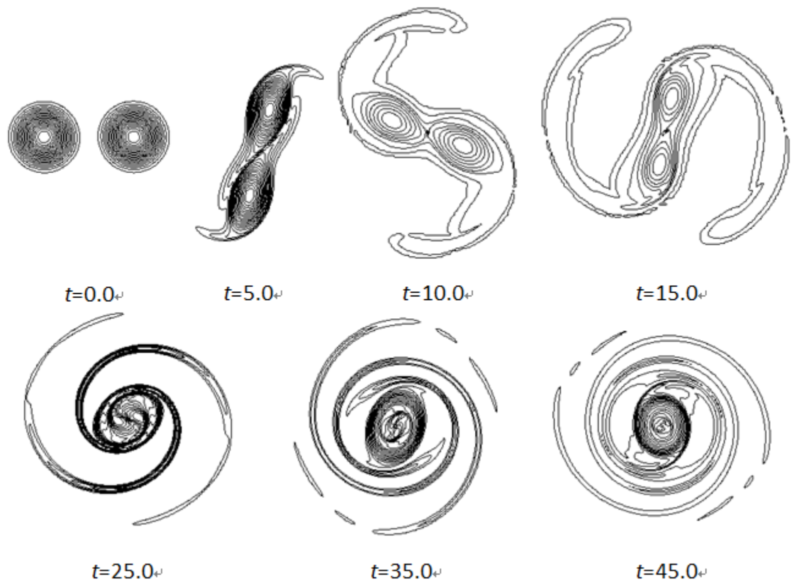
Advanced Panel Method Techniques:

Psuedo-Lagrange multiplier method to solve the ill-posed Neumann problem;

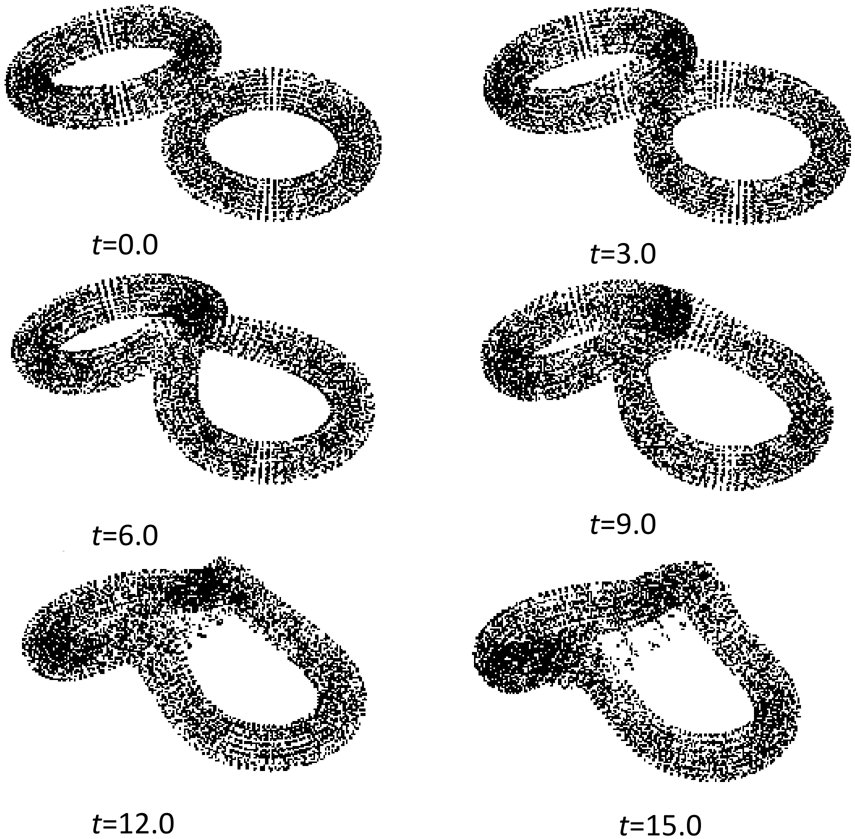
A semi-analytical integral regularization technique to evaluate the singular boundary surface integrals.



Fundamental Applications

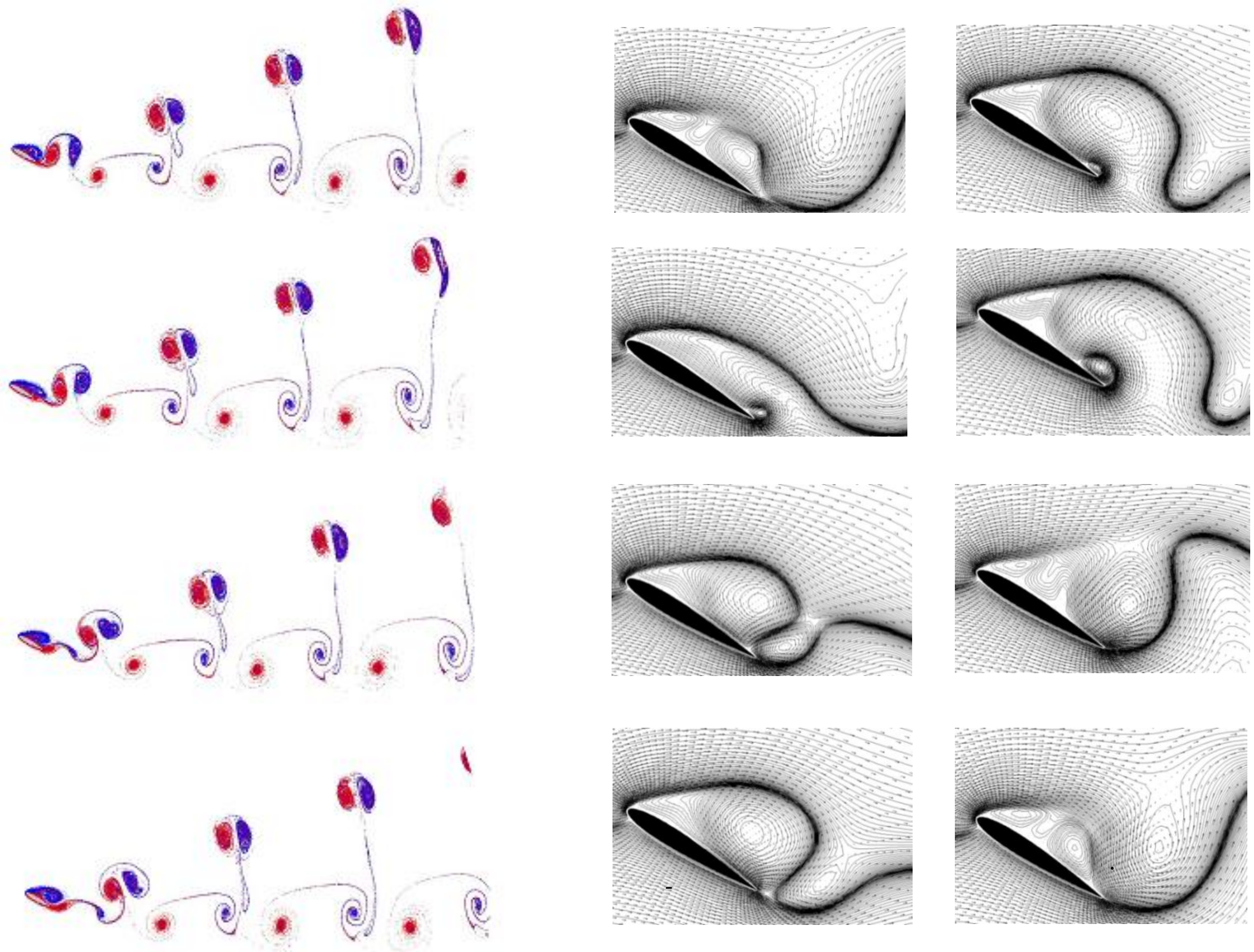


Vortex Merging



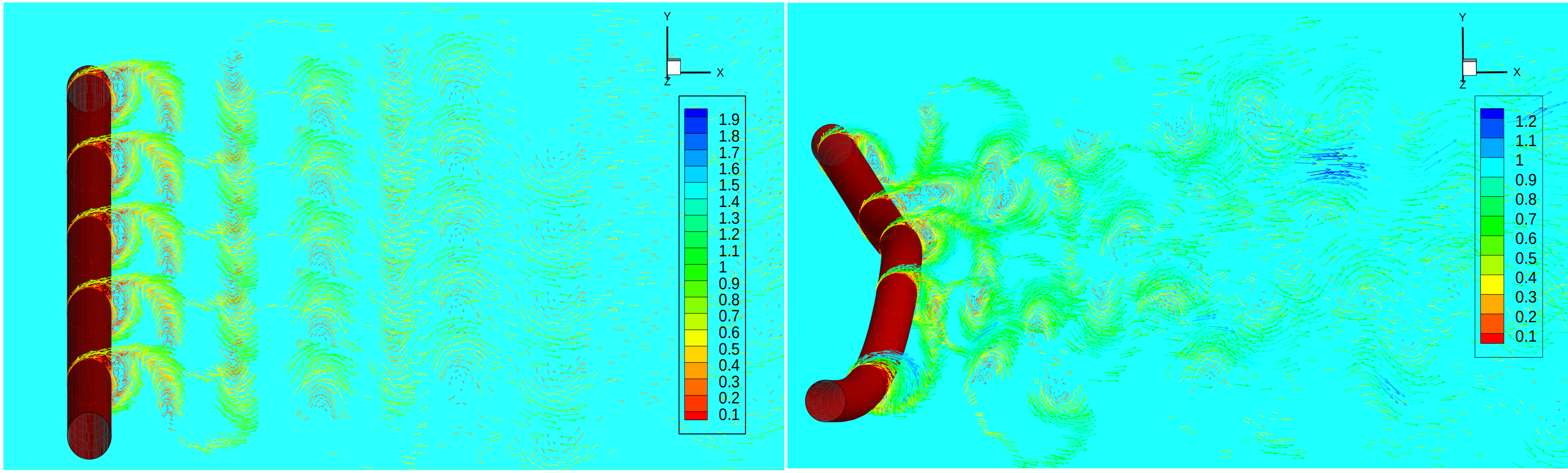
Interaction of Vortex Rings

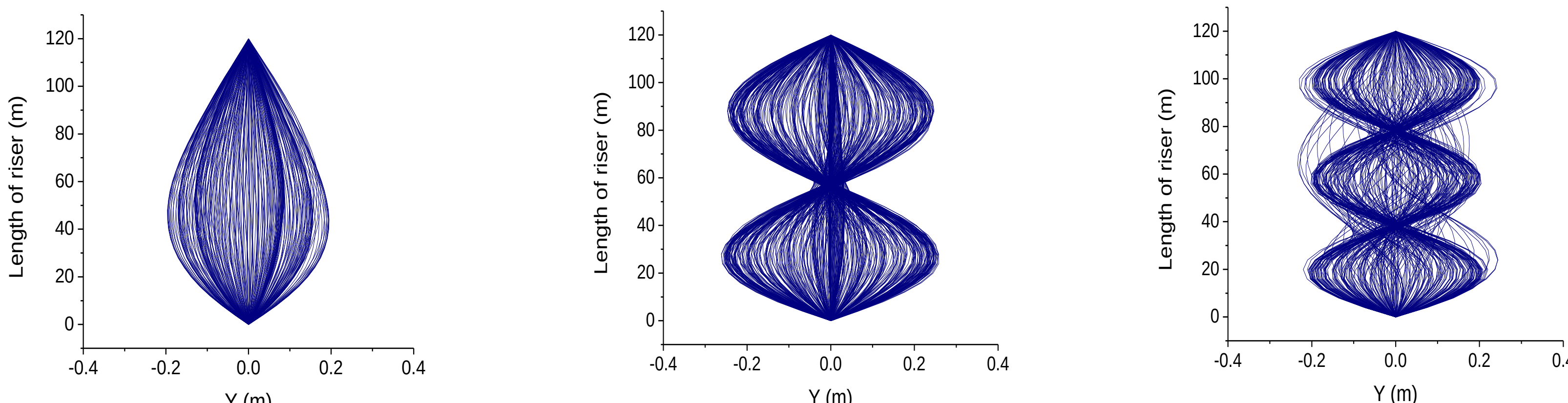
LVM and Applications



Flow Around an Airfoil

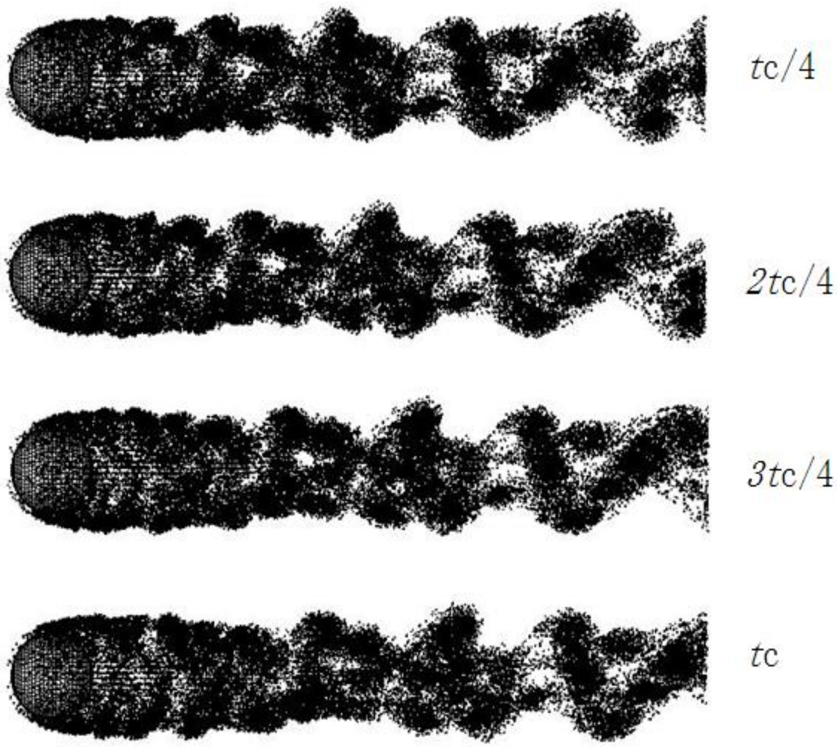
LVM and Applications

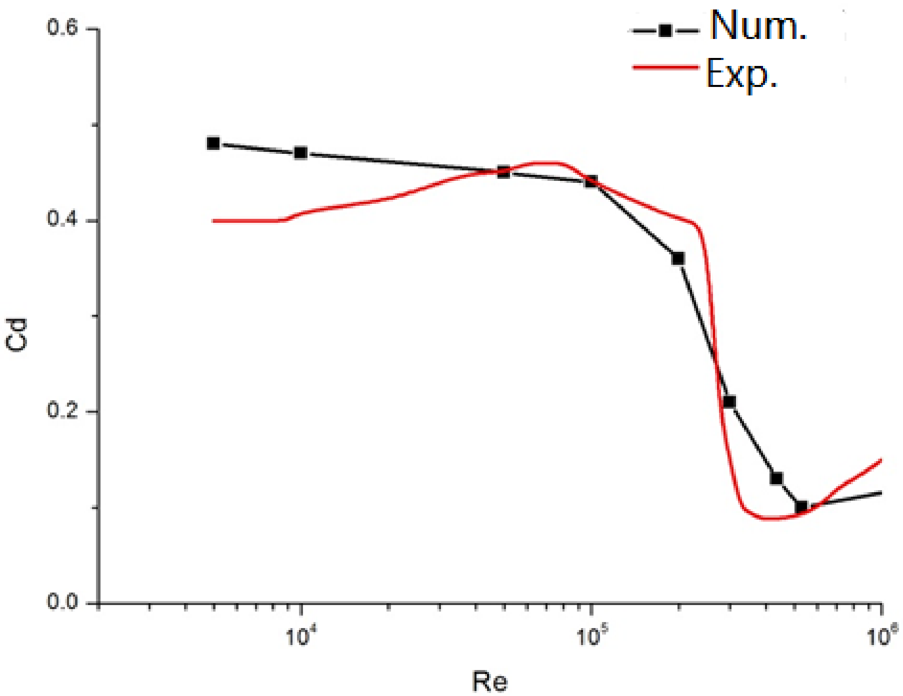




Flow Induced Vibration of Risers

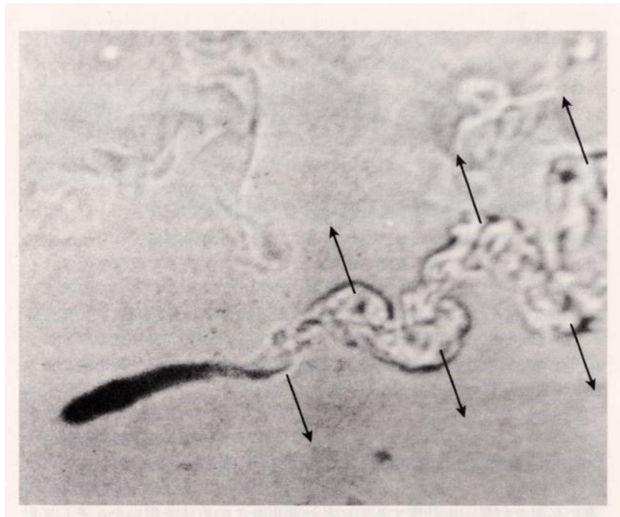
LVM and Applications

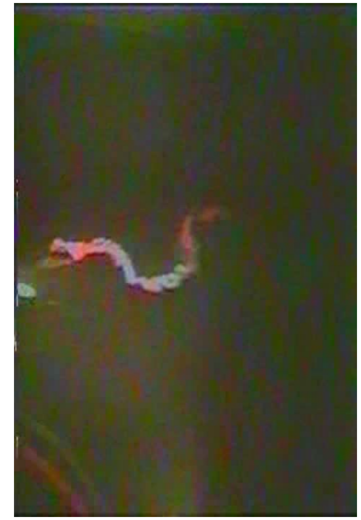


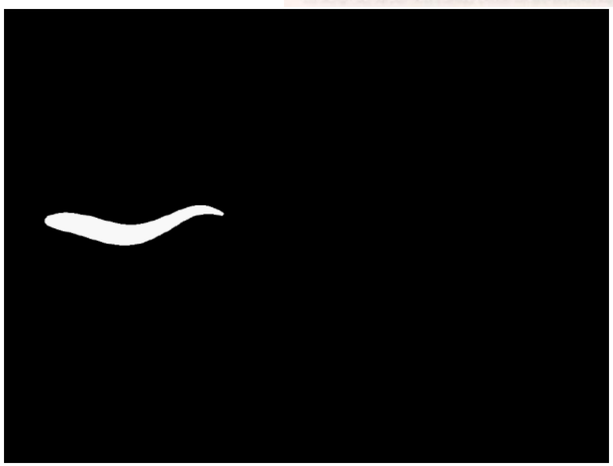


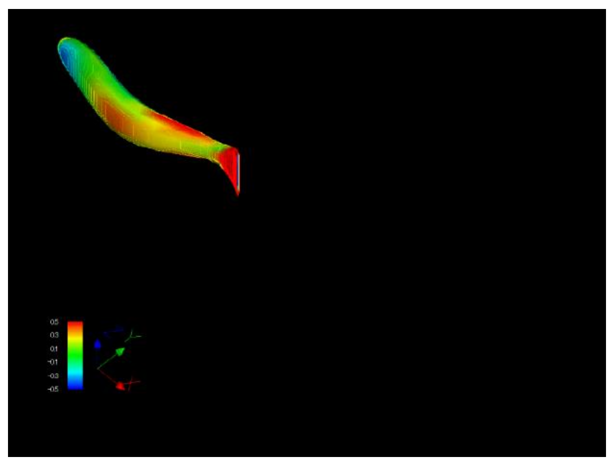
Flow Around a Sphere

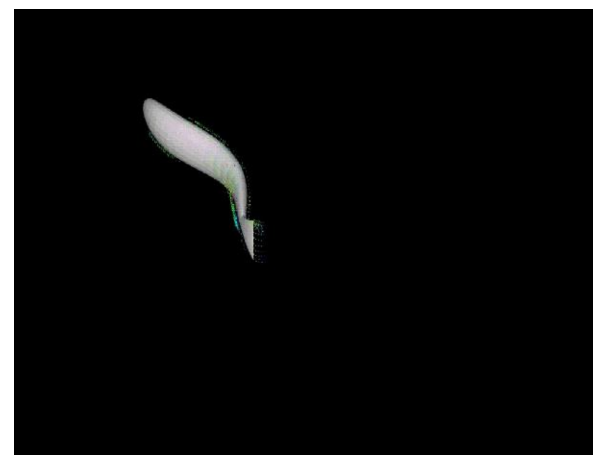
LVM and Applications







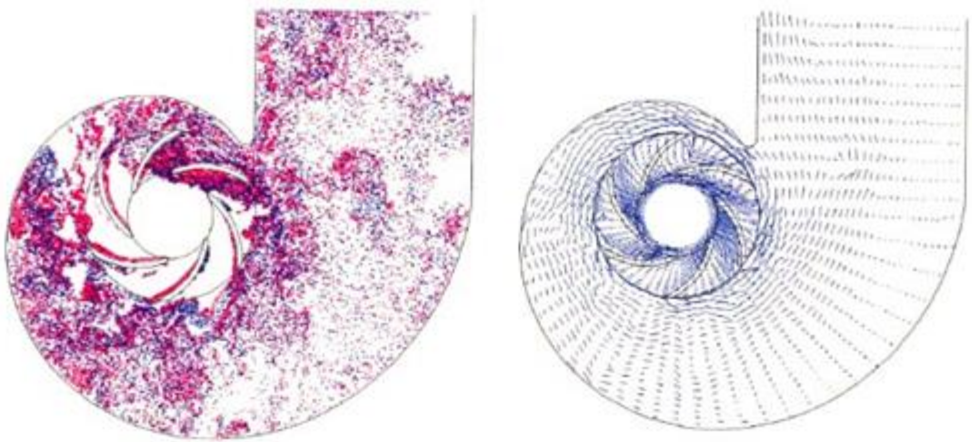




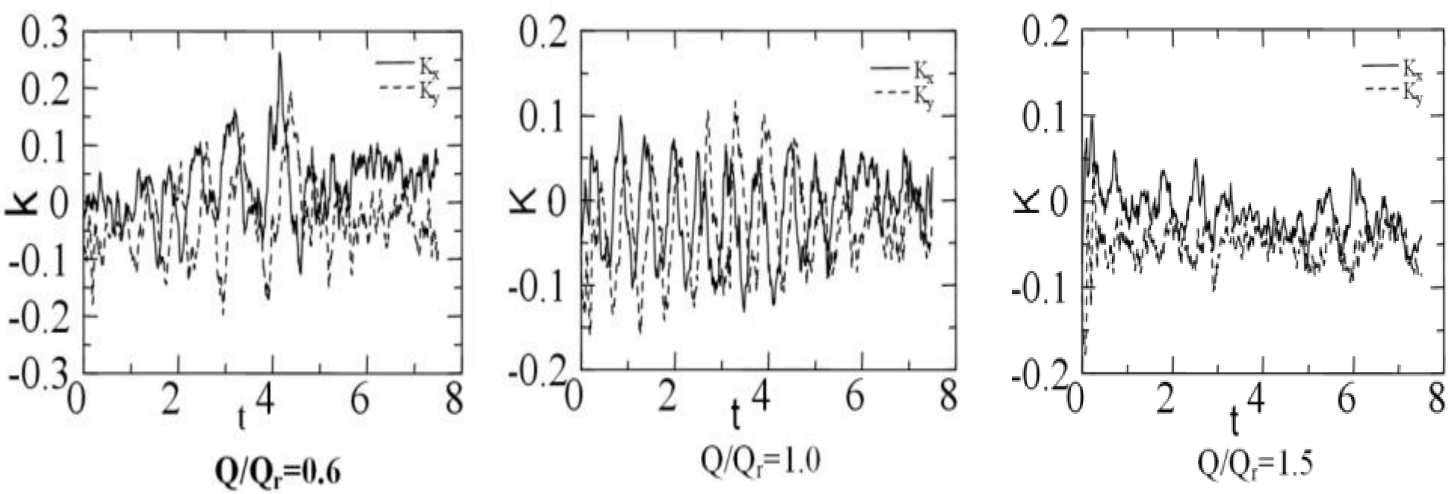
Flow Around Deforming Body

Engineering Applications in Fluid Machinery

Pump

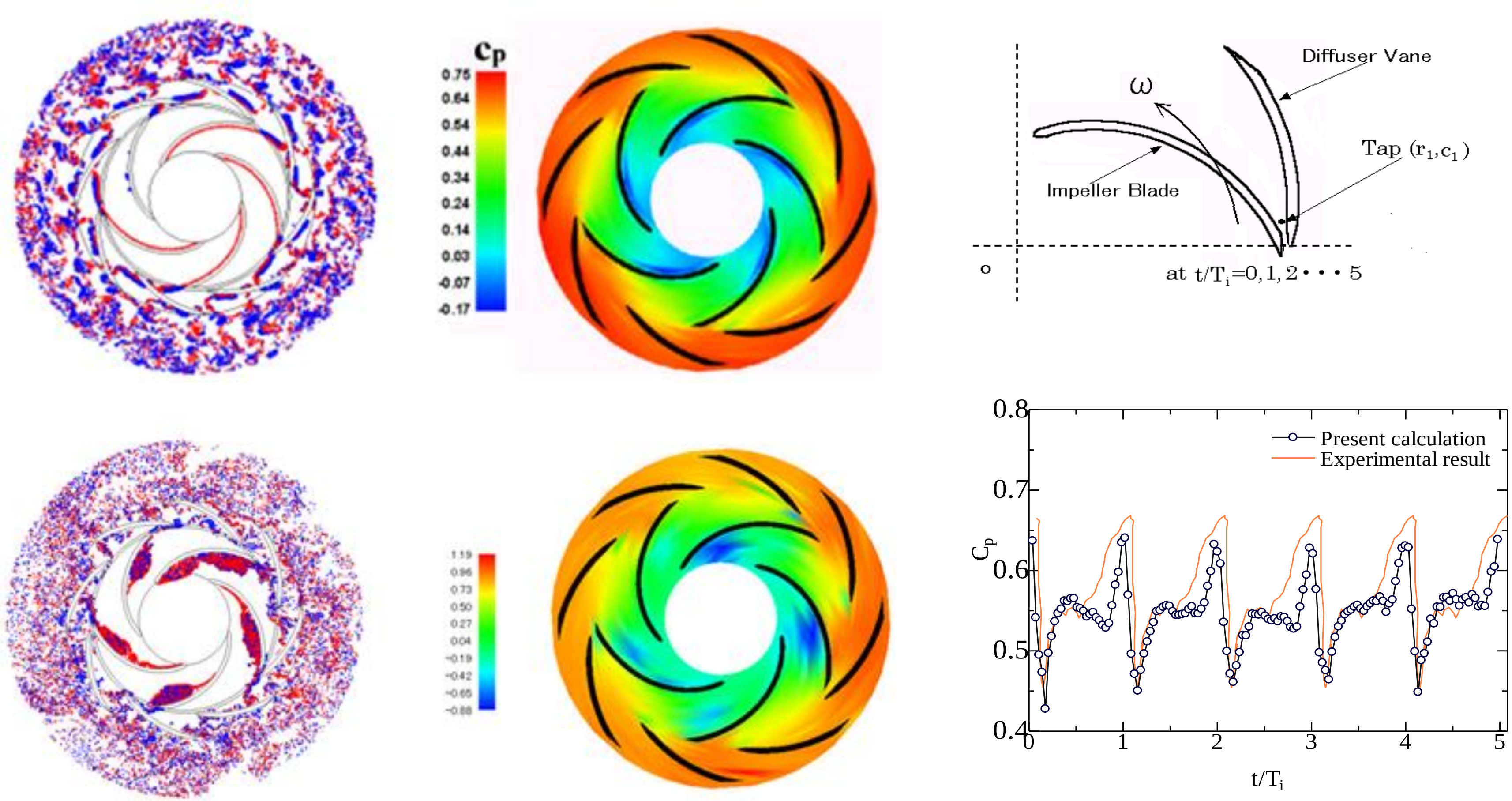


Flow pattern



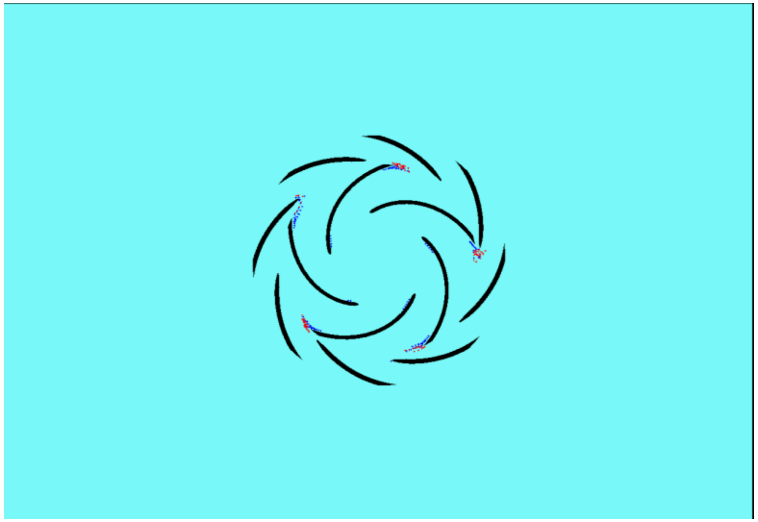
Radial force

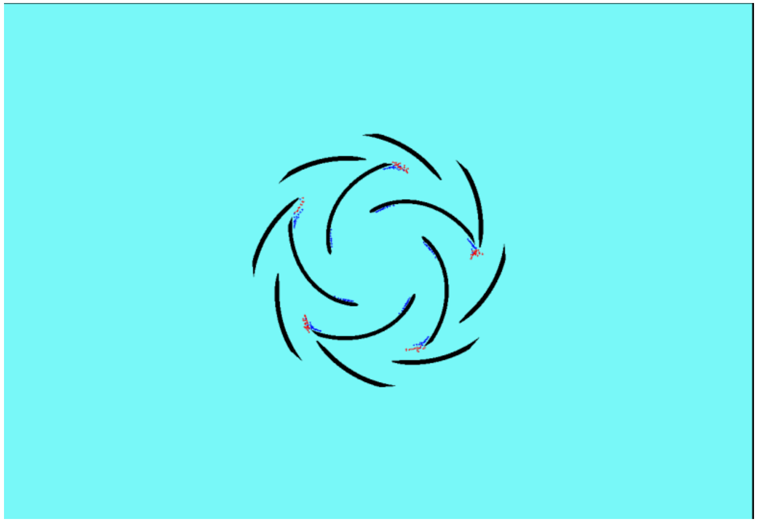
Flow Analysis for Fluid Machinery



Rotor-stator Interaction in Pump

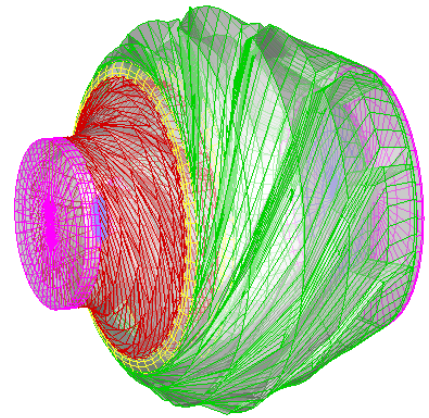
Flow Analysis for Fluid Machinery

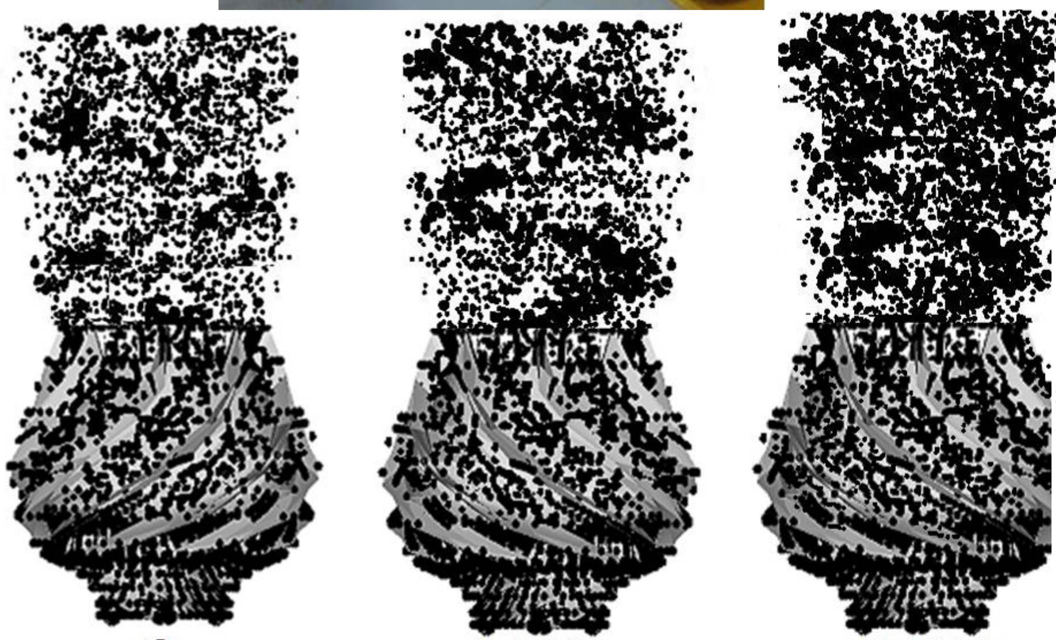




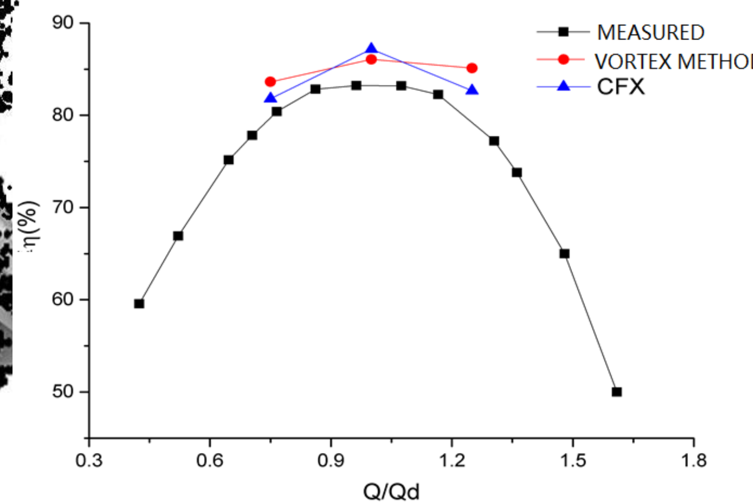
Flow Pattern in Pumps

Flow Analysis for Fluid Machinery





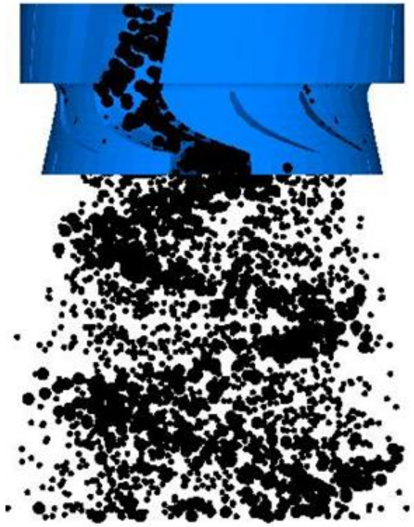
Q_d $1.25Q_d$ $0.75Q_d$



Mixed-flow Pump

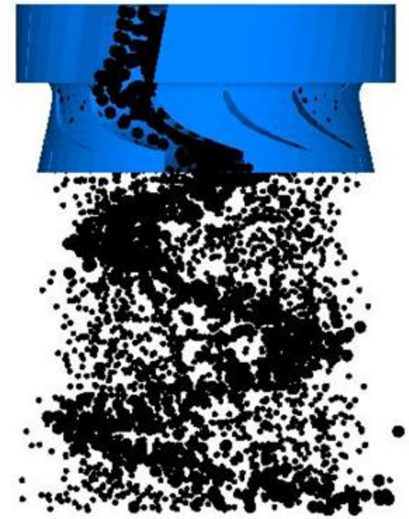
Flow Analysis for Fluid Machinery

Turbine

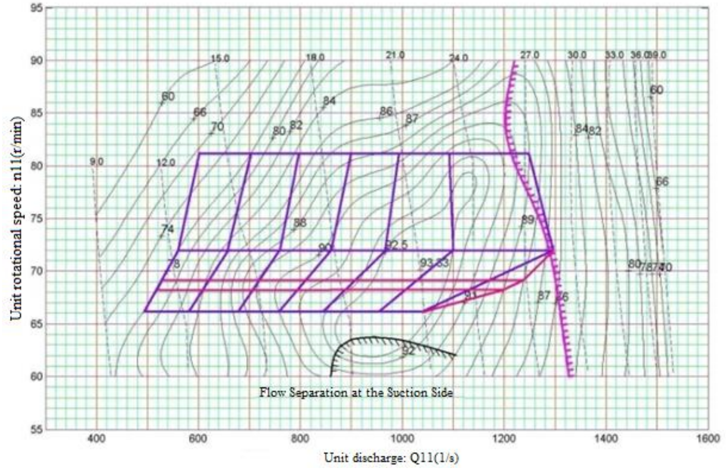


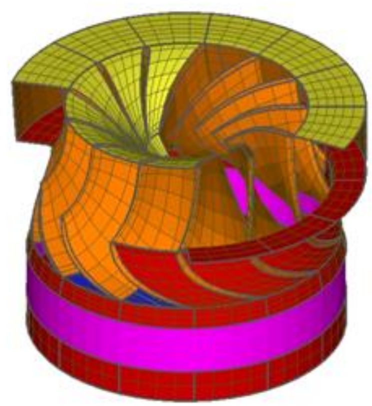
$Q_{11} = 1.04 m^3/s$, $N_{11} = 70.0 rpm$

$Q_{11} = 0.571 m^3/s$, $N_{11} = 65.0 rpm$



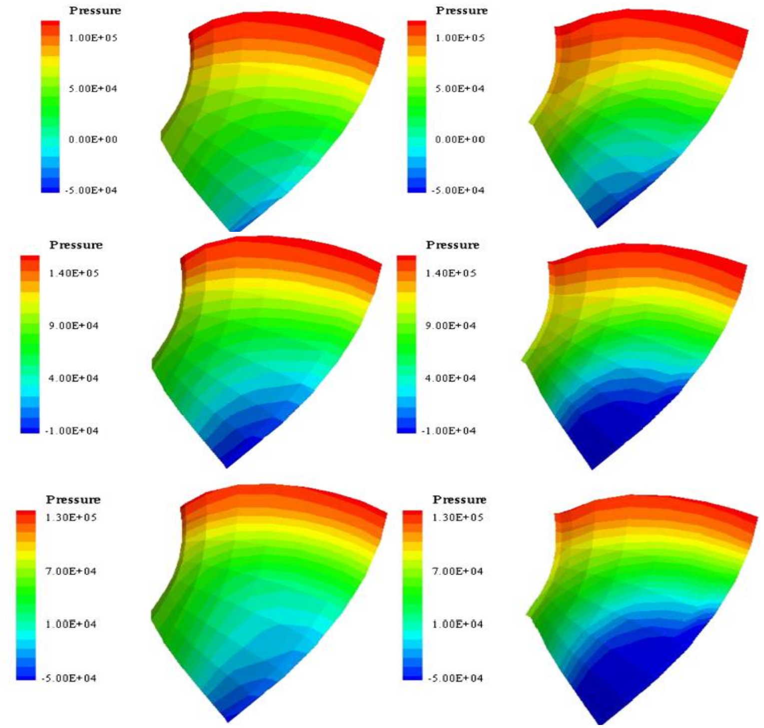
$Q_{11} = 0.571 m^3/s$, $N_{11} = 75.0 rpm$





Flow Analysis for Fluid Machinery

Francis Turbine



condition	$M/(N\cdot\mathrm{m})$	$\eta/\%$	
		calculate	experiment
the design condition	1 020.7	94.0	93.3
the off-design condition 1	622.5	88.6	83.4
the off-design condition 2	485.7	79.7	72.0

➤ Vorticity Dynamics

For incompressible flow, vorticity is generated from solid boundary, boundary vortex flux (BVF) σ,

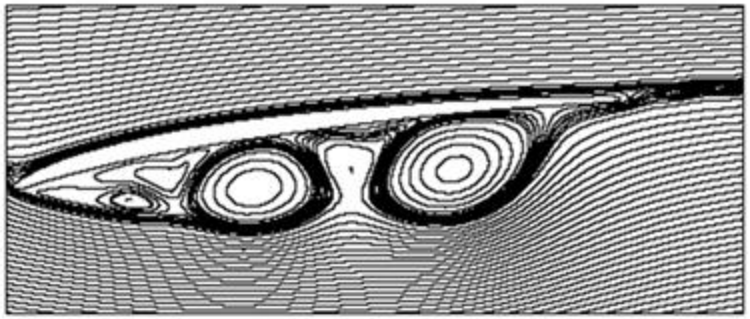
$$\sigma = \nu \frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{n}} = \frac{1}{\rho}\mathbf{n}\times\nabla p + \frac{1}{\rho}\left(\mathbf{n}\times\nabla\right)\cdot\left(\tau\mathbf{n}\right) + \mathbf{n}\times\mathbf{a}_B$$

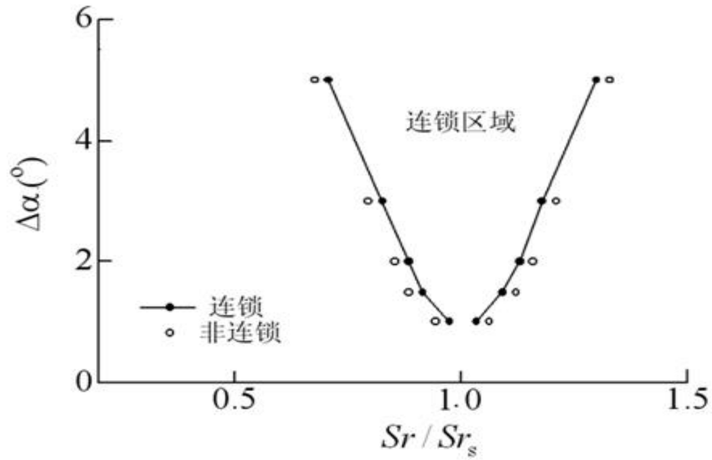
$$\tau = \mu\boldsymbol{\omega}\times\boldsymbol{n}$$

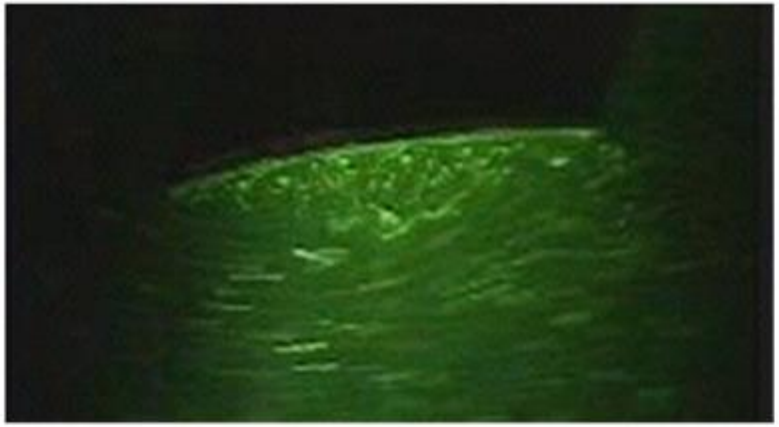
Vorticity and vorticity relative quantity can be used to analyze pressure distribution gradient.

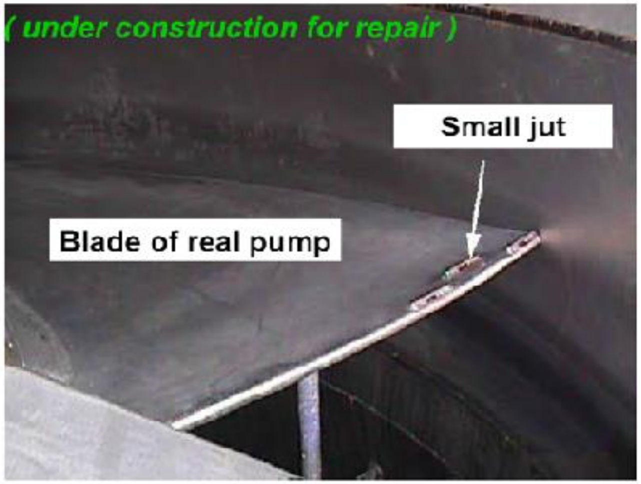
Vortex Flow Control by Optional Design

Passive Control



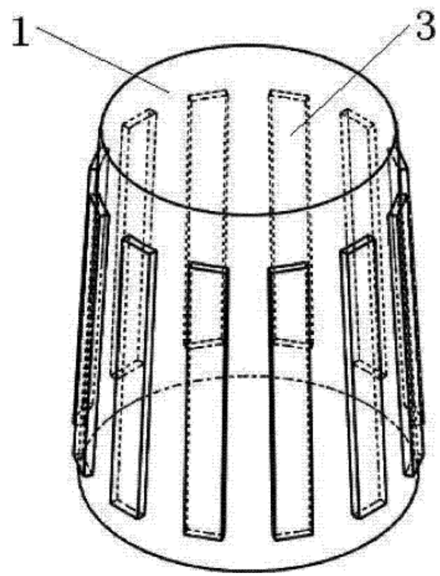


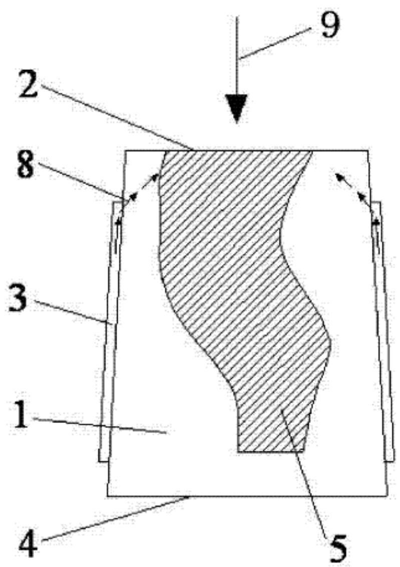


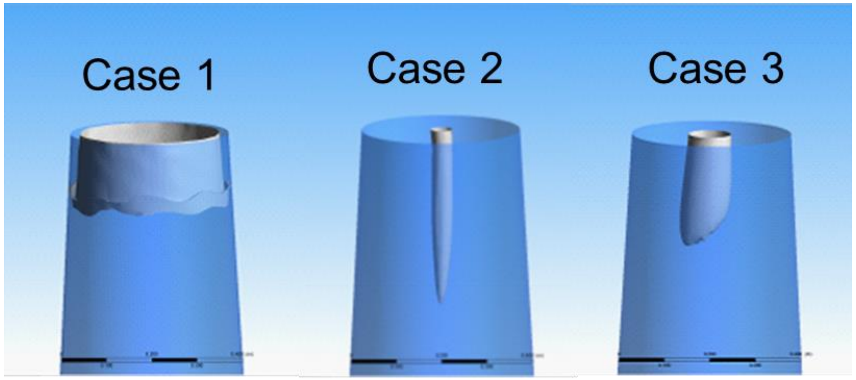


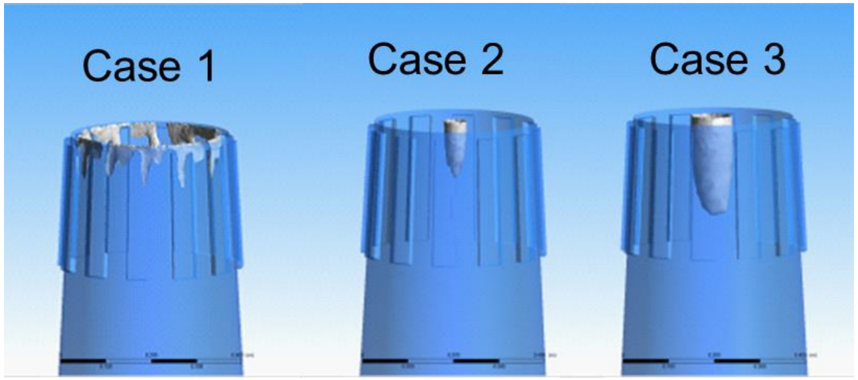
Flow induced vibration of blade

Vortex Flow Control by Optional Design





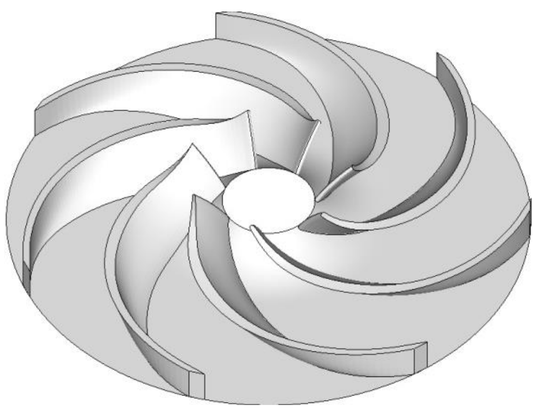




Flow Separation Control by Groove

Vortex Flow Control by Optional Design

- **全三维设计数学模型:用附着涡代替叶片、用源汇代替有限厚度对流场的作用。根据流动的周期性，将转轮内部流动分解成周向平均流动和周期脉动流动**
- **影响转轮性能的参数包括：轴面流道形状、叶片数目、叶片进出口位置及倾角和速度矩分布等。采用多目标遗传算法对速度矩分布和叶片进出口位置及倾角等进行优化。**



全三维设计

平均流动方程：

$$\overline{W}_r=\overline{V}_r=-\frac{1}{r}\frac{\partial\Psi}{\partial z}+\frac{\partial\Phi}{\partial r}$$

$$\overline{W}_z=\overline{V}_z=\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{\partial\Phi}{\partial z}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Psi}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial\Psi}{\partial z}\right)=-\frac{\partial\left(\overline{V}_\theta r\right)}{\partial r}\frac{\partial f}{\partial z}+\frac{\partial\left(\overline{V}_\theta r\right)}{\partial z}\frac{\partial f}{\partial r}+\frac{1}{\overline{W}^2}\left[\frac{\partial\overline{E}_r}{\partial r}\left(\overline{W}_z+\overline{W}_\theta\frac{r\partial f}{\partial z}\right)-\frac{\partial\overline{E}_r}{\partial z}\left(\overline{W}_r+\overline{W}_\theta\frac{r\partial f}{\partial r}\right)\right]$$

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{\partial^2\Phi}{\partial z^2}=\frac{B}{2\pi r}\left[\frac{\partial}{\partial r}\left(t_\theta\overline{V}_r\right)+\frac{\partial}{\partial z}\left(t_\theta\overline{V}_z\right)\right]$$

脉动流动方程：

$$\frac{\partial^2\phi_k^c}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_k^c}{\partial r}+\frac{\partial^2\phi_k^c}{\partial z^2}-\frac{k^2B^2}{r^2}\phi_k^c=2\cos(kBf)\overline{K}-2\cos(kBf)\left(\frac{\partial f}{\partial r}\frac{\partial\left(\overline{V}_\theta r\right)}{\partial r}+\frac{\partial f}{\partial z}\frac{\partial\left(\overline{V}_\theta r\right)}{\partial z}\right)-\frac{2}{kB}\sin(kBf)\left(\frac{\partial^2\left(\overline{V}_\theta r\right)}{\partial r^2}+\frac{1}{r}\frac{\partial\left(\overline{V}_\theta r\right)}{\partial r}+\frac{\partial^2\left(\overline{V}_\theta r\right)}{\partial z^2}\right)$$

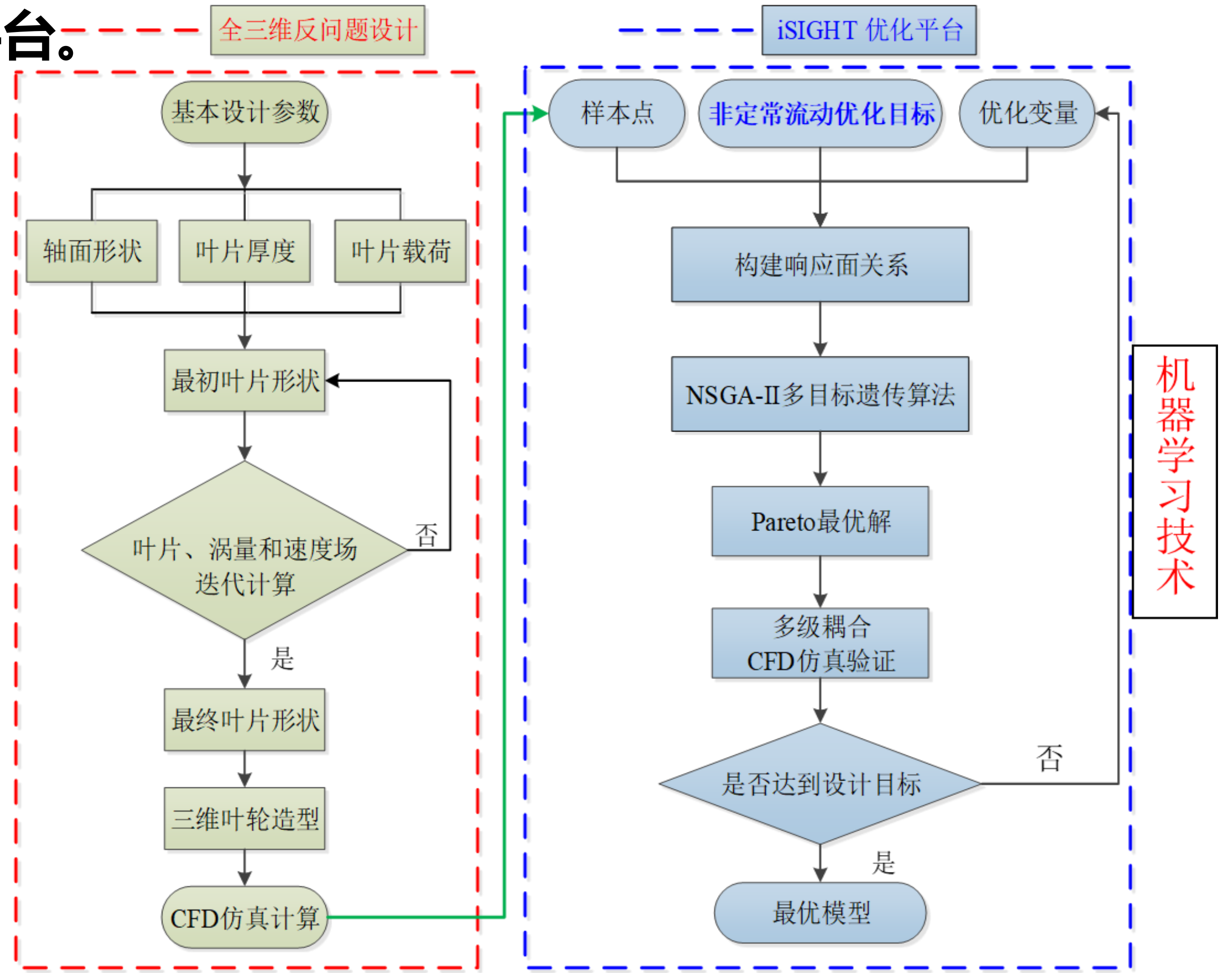
$$\frac{\partial^2\phi_k^s}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_k^s}{\partial r}+\frac{\partial^2\phi_k^s}{\partial z^2}-\frac{k^2B^2}{r^2}\phi_k^s=2\sin(kBf)\overline{K}-2\sin(kBf)\left(\frac{\partial f}{\partial r}\frac{\partial\left(\overline{V}_\theta r\right)}{\partial r}+\frac{\partial f}{\partial z}\frac{\partial\left(\overline{V}_\theta r\right)}{\partial z}\right)+\frac{2}{kB}\cos(kBf)\left(\frac{\partial^2\left(\overline{V}_\theta r\right)}{\partial r^2}+\frac{1}{r}\frac{\partial\left(\overline{V}_\theta r\right)}{\partial r}+\frac{\partial^2\left(\overline{V}_\theta r\right)}{\partial z^2}\right)$$

叶片方程：

$$\mathrm{d}f=\frac{\overline{V}_\theta r+\tilde{V}_\theta r-\Omega r^2}{r^2\overline{V}_m}\mathrm{d}s$$

Vortex Flow Control by Optional Design

- 流动分析、全三维设计、以及多目标优化方法结合，开发流体机械流动分析和优化设计平台。



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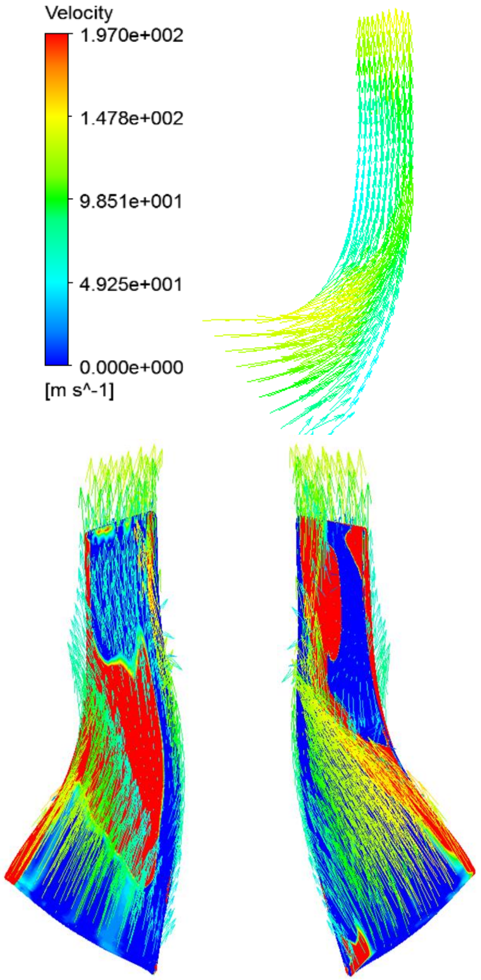

N-S Eq. in rotating frame:

$$\frac{D\vec{W}}{dt} + 2\vec{\omega}\times\vec{W} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) = \vec{f} - \nabla(\frac{p}{\rho}) + \nu\Delta\vec{V}$$

Streamwise component of absolute vorticity：

Secondary flow

$$\vec{W}\cdot\nabla(\vec{W}\cdot\vec{\Omega}) - 2\vec{\Omega}\cdot\nabla(\vec{W}\cdot\vec{W}) = \vec{\Omega}\cdot\left(2\vec{\omega}\times\vec{W}\right)$$

Velocity on meridional surface and blade surface for inviscid flow

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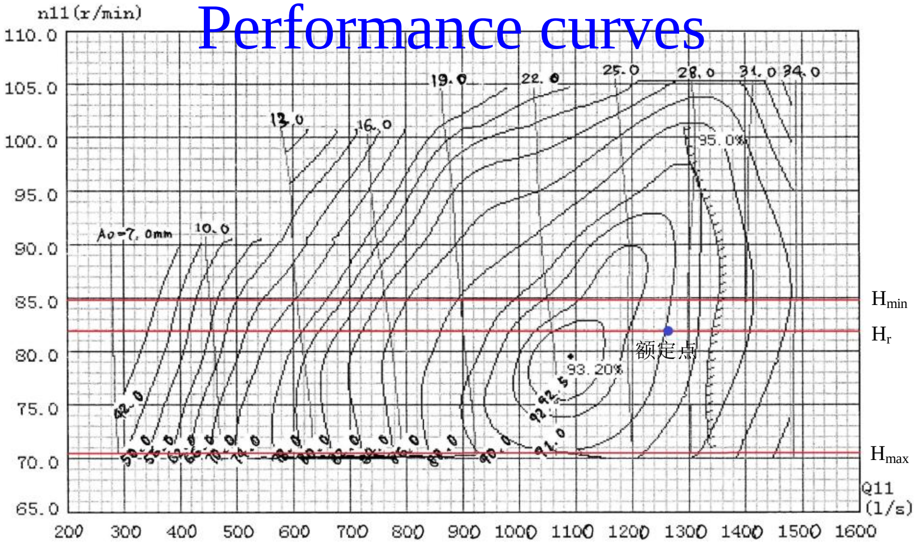
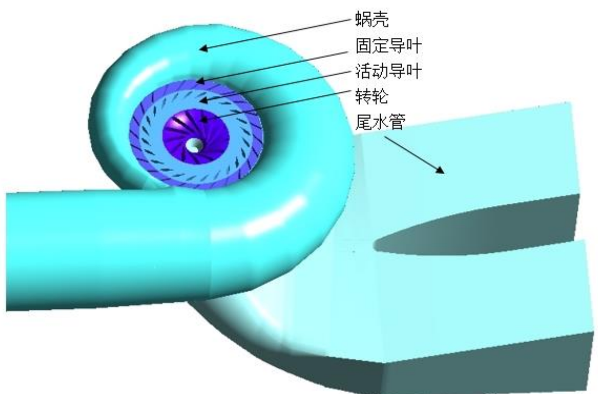
Turbine Runner（Francis Turbine HLA551）

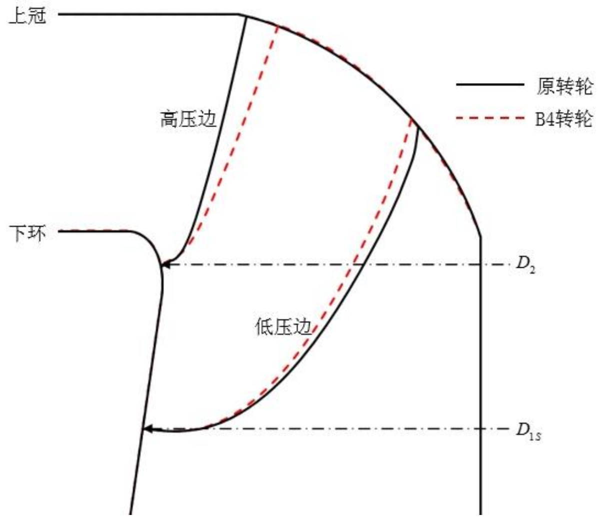
Performance curves

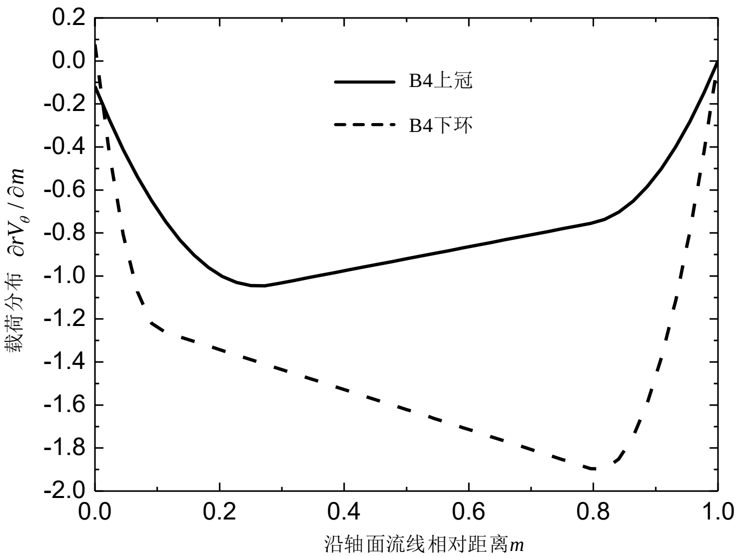


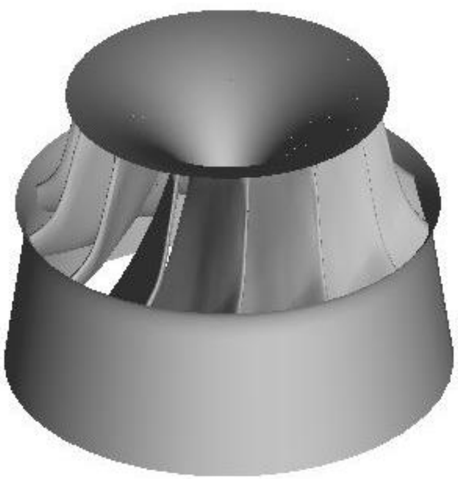


Schematic of turbine

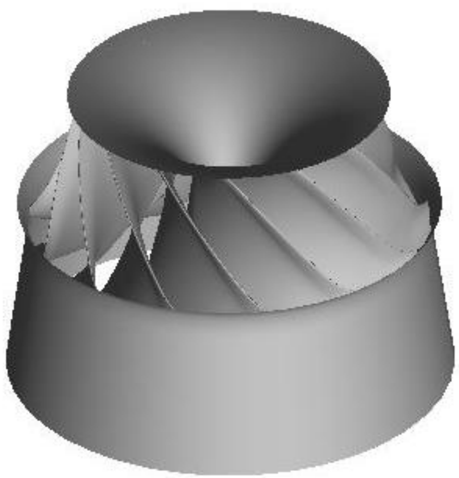
Vortex Flow Control by Optional Design







Initial runner



Optimized runner

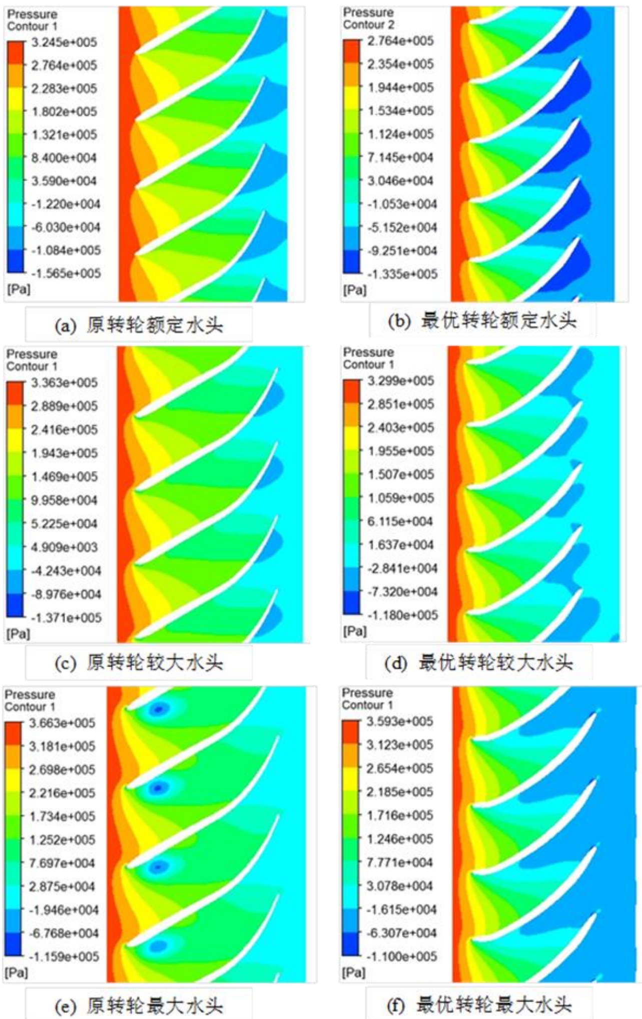
Efficiency improvement:

Hr: 0.91%

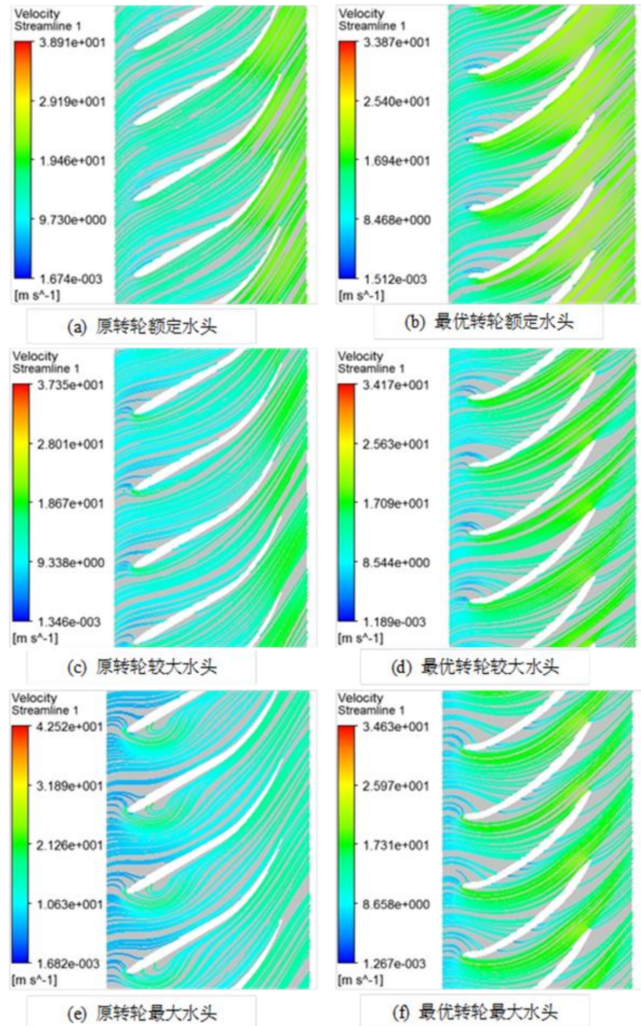
Hmax: 0.37%

Reduction of Lowest Pressure: 376.2KPa

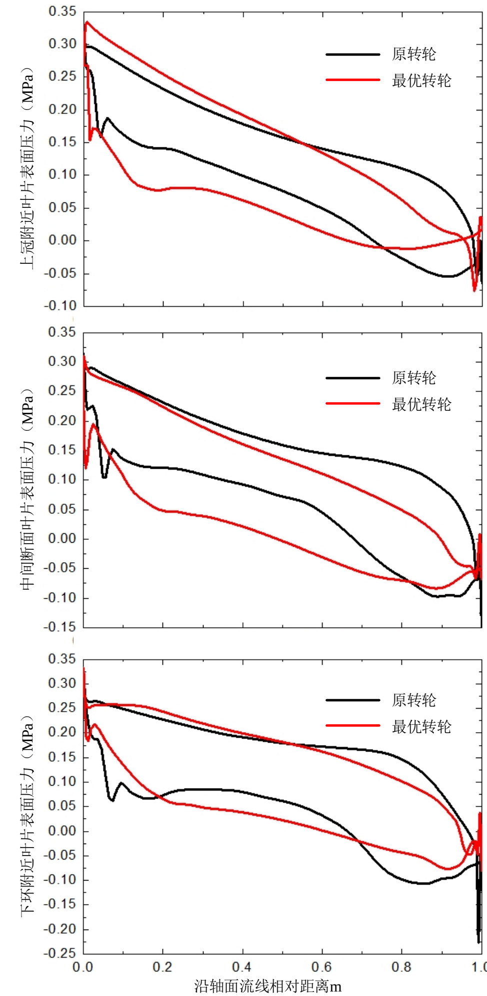
Vortex Flow Control by Optional Design



Pressure distribution



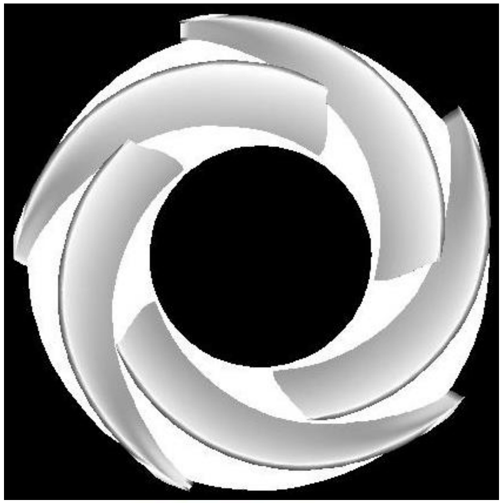
Streamlines

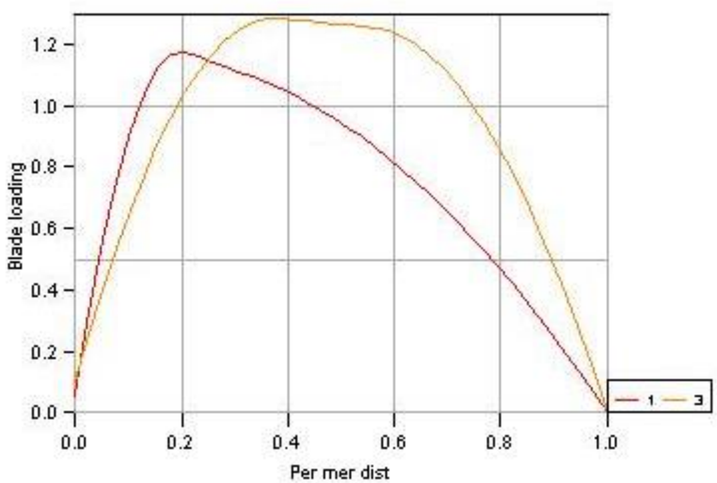


Pressure difference

Vortex Flow Control by Optional Design

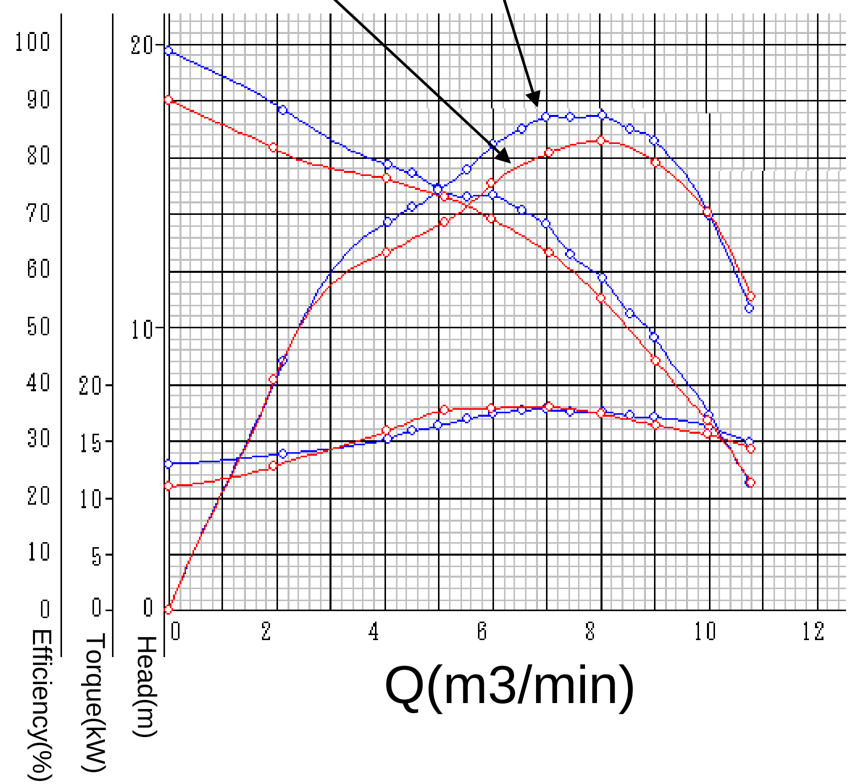
Pump Impeller（Mixed Flow Pump Ns=140）



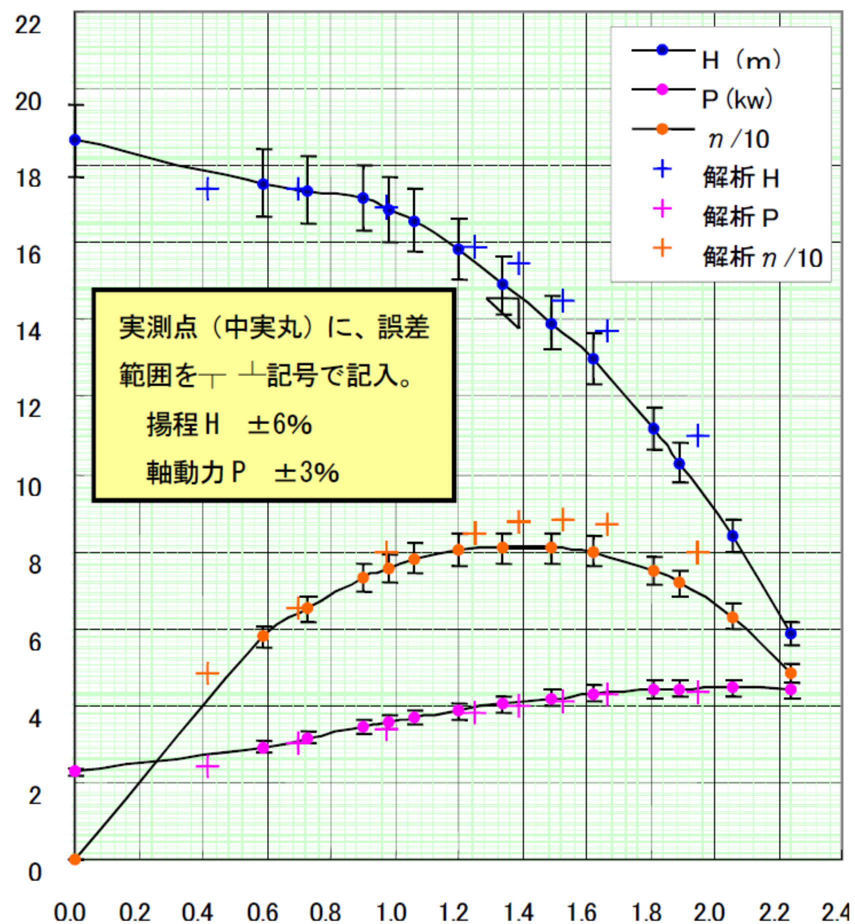


Red: non-optimized impeller

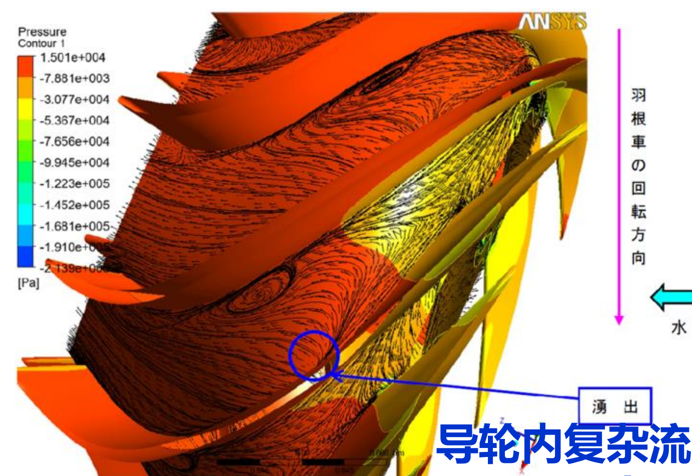
Blue:optimized impeller



Vortex Flow Control by Optional Design



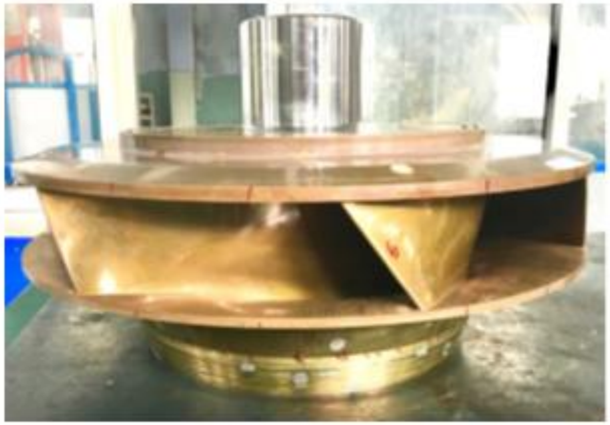
混流泵流动计算



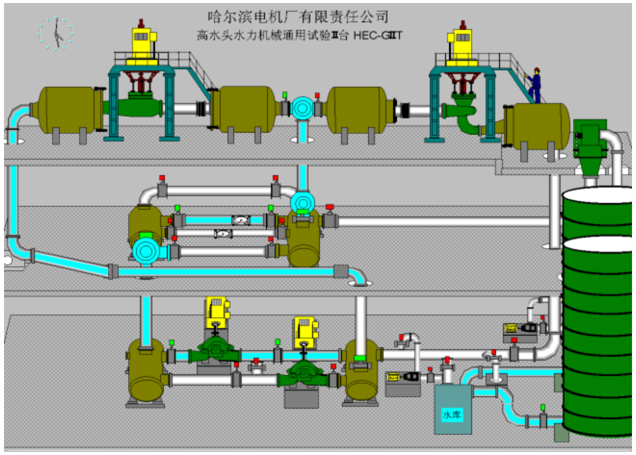
导轮内复杂流动
(50%Qd工况)

Vortex Flow Control by Optional Design

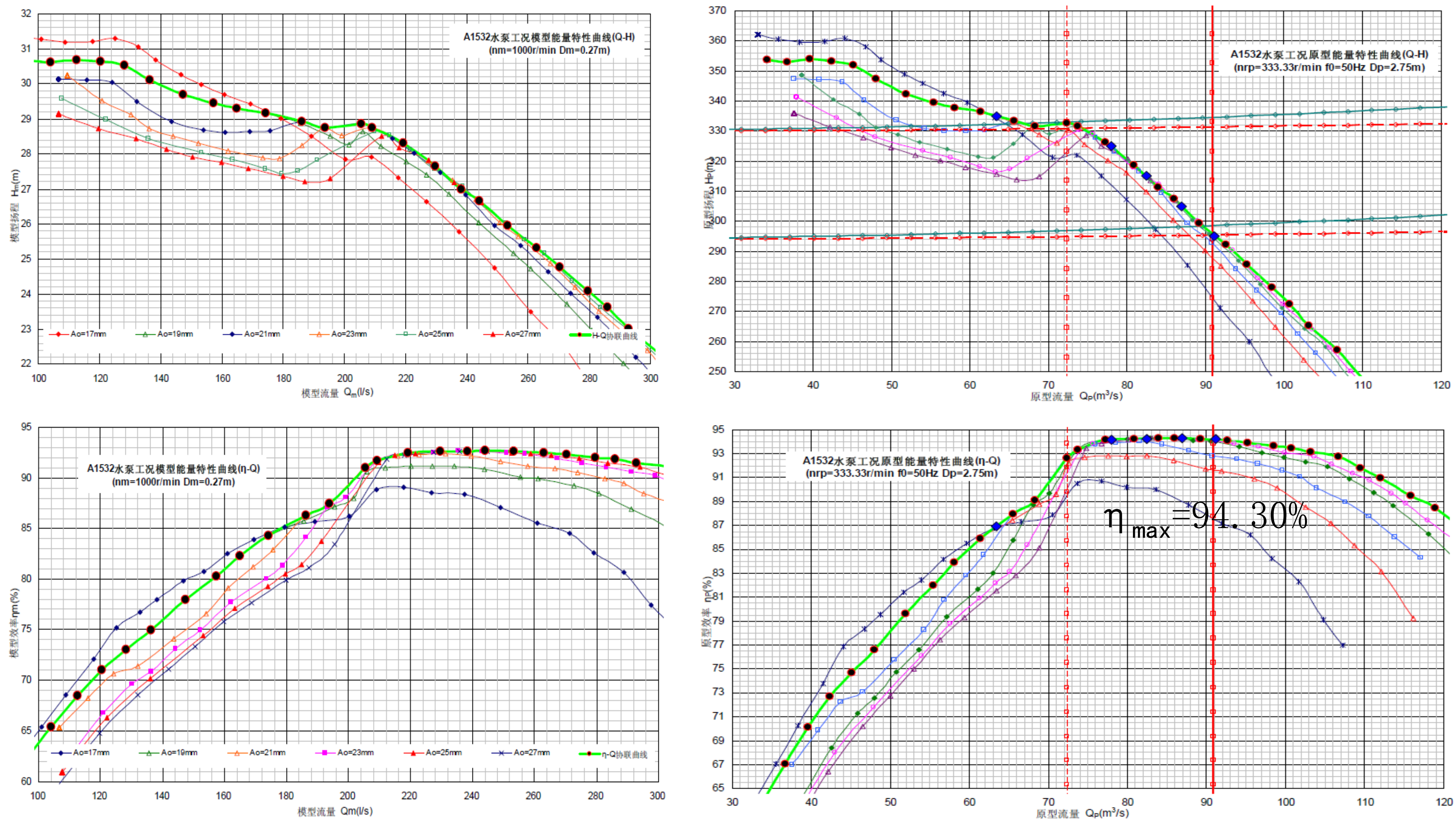
Middle-Head Runner-Model



Optimized runners

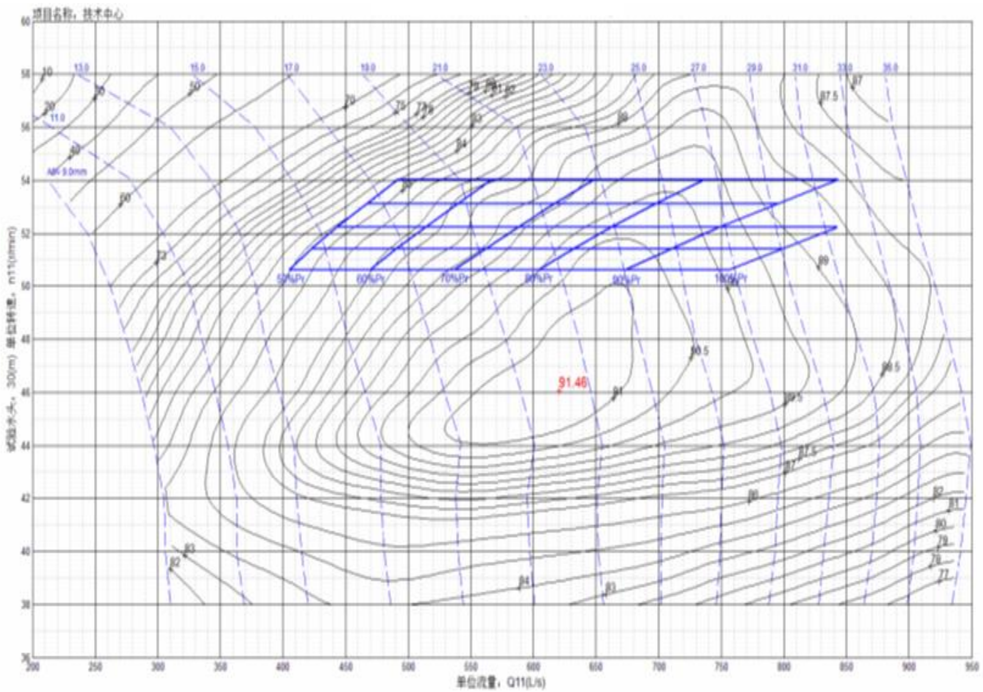


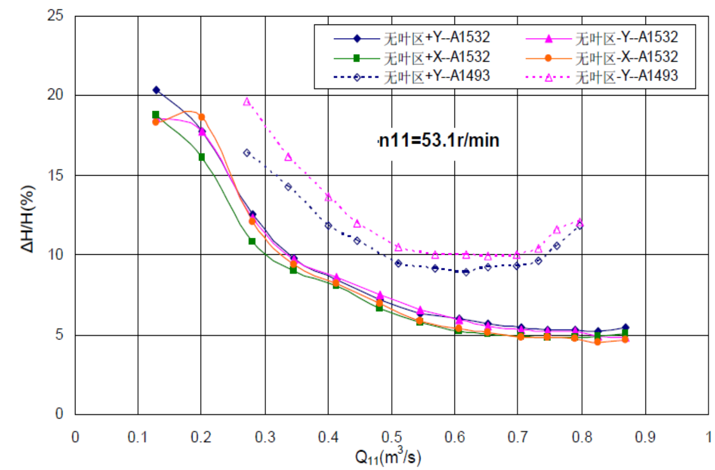
Vortex Flow Control by Optional Design

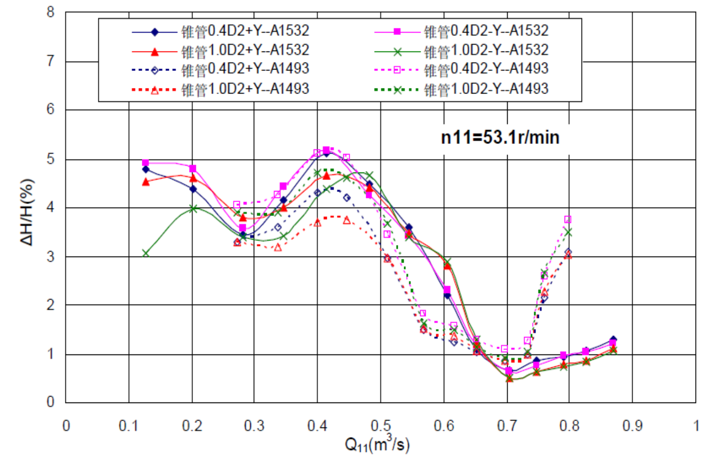


The measured performance in pump mode

Vortex Flow Control by Optional Design

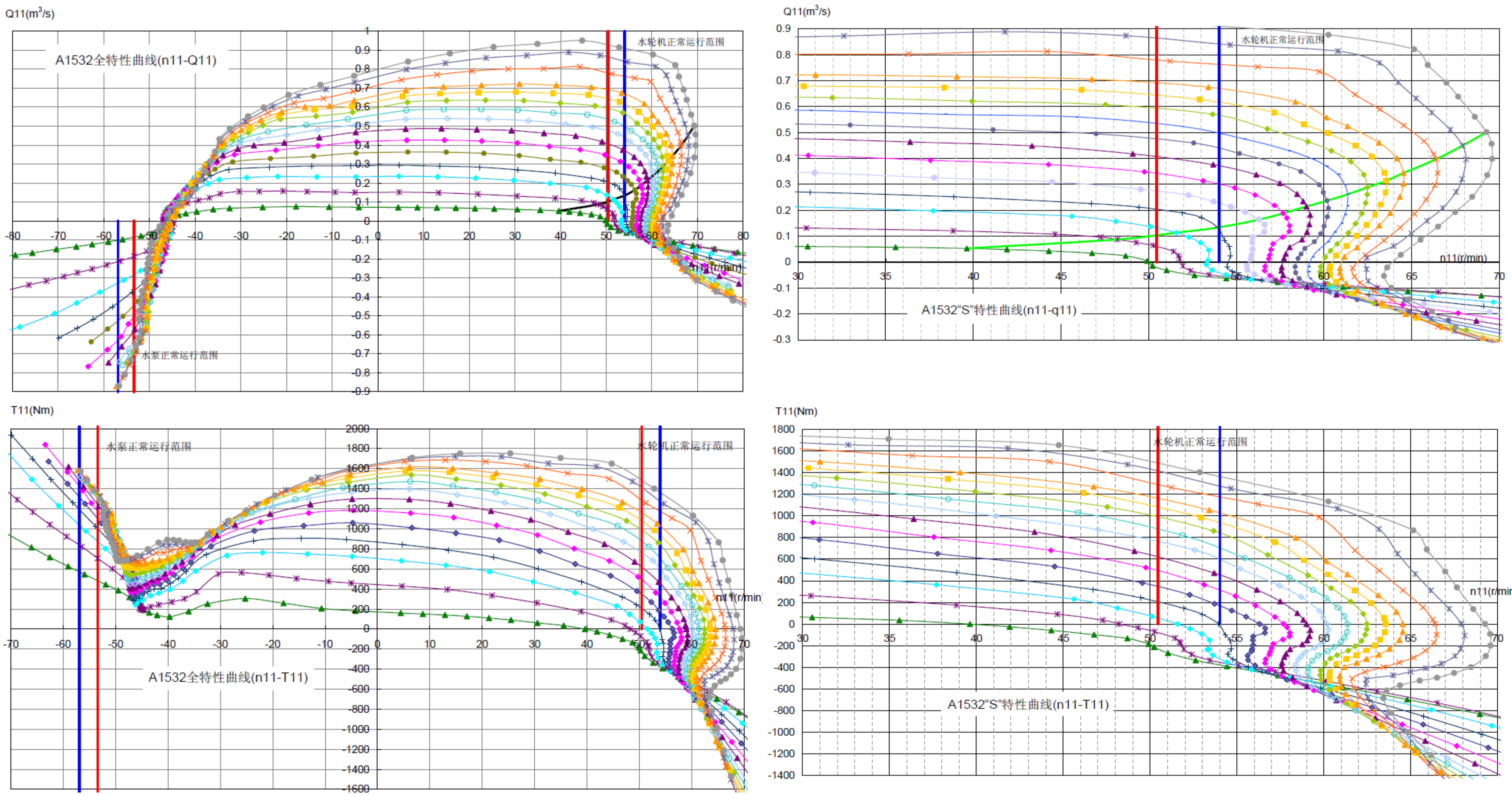






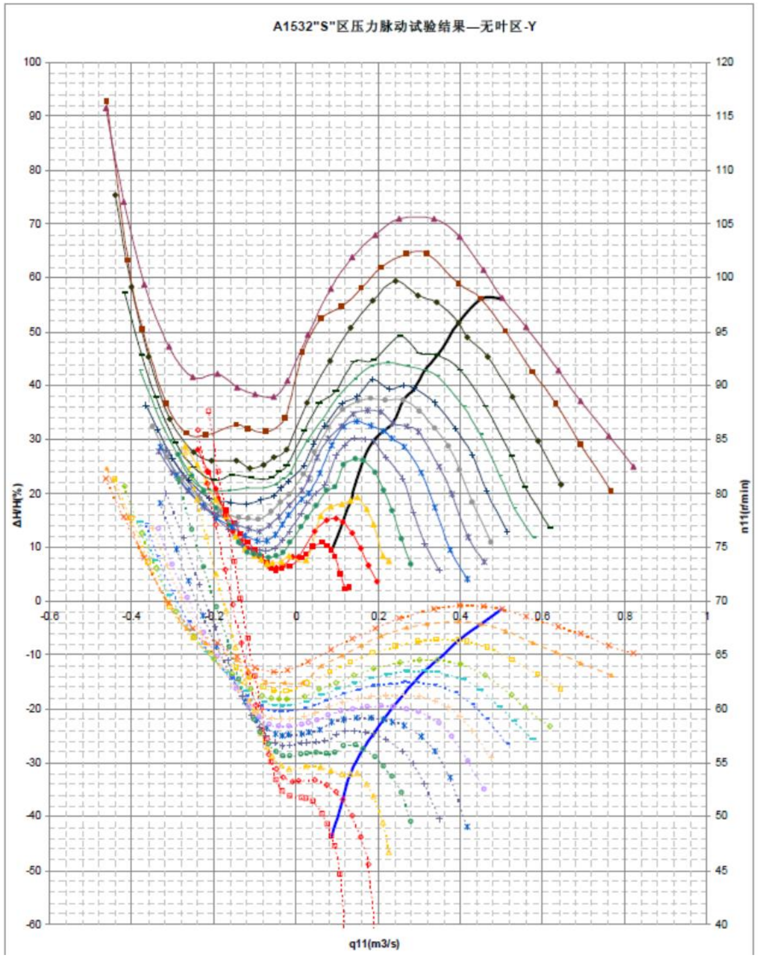
The measured performance in turbine mode

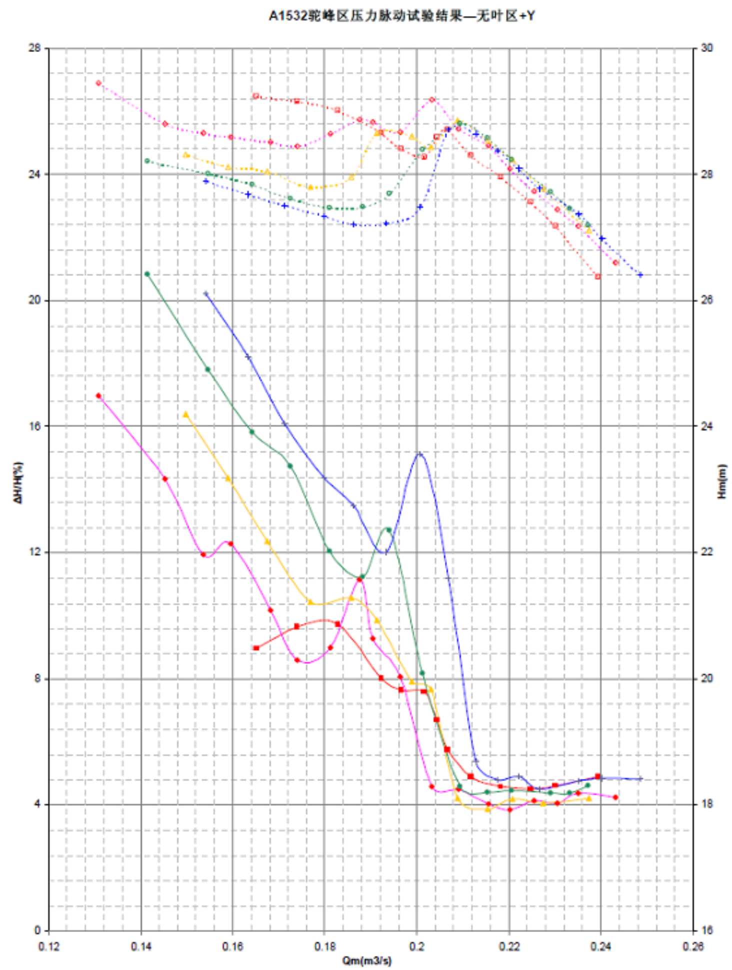
Vortex Flow Control by Optional Design



The measured complete characteristics

Vortex Flow Control by Optional Design





The measured pressure fluctuations in S-region and hump region

Summary

- The numerical treatment and the application results show that the Lagrangian vortex method is useful for study of complex flows of moving boundary problems in various fields of engineering and science.

- Based on physics of flow, vorticity and vorticity dynamics has been used in fluid machinery for flow analysis and flow diagnosis.

- Vortex stability and vortical flow control should be deeply investigated for fluid machinery based on optimization technique.

Summary

➢ 流动分析(CFD Techniques)

① CFD techniques are indispensable;

② Eulerian-based schemes are usually used;

③ Effects and workload of mesh density;

④ Turbulence models, no universal model.

➢ 优化设计(Optimization Design)

① Inviscid-viscid interaction;

② Relationship between hydraulic and geometric parameters with outer performance;

③ No direct relationship between inner flow patterns with outer performance;

④ Difficult in considering unsteady flow characteristics.

水力设计、结构设计成熟固化；企业可应用！大学研究？

Thank You Very Much For Your Attentions!