

岩土工程大变形数值模拟的 光滑粒子有限单元法

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张巍







□ 岩土工程领域经常涉及大变形问题 { 滑坡、泥石流 □ 岩土工程领域经常涉及大变形问题 { 取样、原位试验、预制桩贯入



云南省镇雄赵家沟村(2013年, 46人死亡)



浙江省丽水市(2015年, 38人死亡)



• 12·20深圳山体滑坡(2015年,73人死亡)



● 原位试验





● 预制桩贯入









□ Why PFEM (particle finite element method)?

We have a lot of numerical methods that can solve large deformation problems, i.e. ALE, SPH, CEL, MPM, PFEM There are mainly two reasons:

- Easy to implement, it can be easy realized from FEM codes.
- It inherits the accuracy of FEM.



D PFEM applications:

Granular columns collapse





Progressive landslide



Cone penetration test



□ Typical step of PFEM:

1. On the basis of cloud of particles, the Delaunay triangulation technique is used to build the FEM mesh.

2. The alpha-shape approach is used to identify the entire problem domain.

3. Map the state variables (strain, stress, etc.) from particles to Gauss points.

4. Solve the governing equations via a standard incremental FEM approach.

5. Map the state variables (strain, stress, etc.) from the Gauss points to the particles

6. Modify the positions of particles and transfer all the field information of particles to form a new cloud of particles.

7. Go back to Step 1 and repeat until the problem-dependent stop condition.



D PFEM defects:

• Mapping induced errors due to the frequent information (e.g. stress, strain, plastic strain)

transfer between old and new meshes.

• Volumetric locking due to the use of low order elements, and therefore 6-node elements are generally used.



□ Core of SPFEM:

Strain smoothing technique for nodal integration is incorporated.

Construction of strain smoothing cells associated with particles



Details in:

Liu, G. R., Nguyen-Thoi, T., Nguyen-Xuan, H., and Lam, K. Y., 2009. A node-based smoothed finite element method (NS-FEM) for upper bound solutions to solid mechanics problems. Computers and Structures, 87(1-2), 14-26.

Chen, J. S., Wu, C. T., Yoon, S., and You, Y. (2001). "A stabilized conforming nodal integration for Galerkin mesh-free methods." Int. J. Numer.Methods Eng., 50(2), 435–466.

Strain smoothing technique for SPFEM

The smoothed derivative of shape function for particle k at the h direction (h = x, y) can be obtained as follows:

$$\widetilde{b}_{lh}(\boldsymbol{x}_k) = \frac{1}{A^{(k)}} \int_{\Gamma^{(k)}} N_l(\boldsymbol{x}) n_h(\boldsymbol{x}) d\Gamma$$

As the gradient of displacement is constant in each element (3-node triangular element), $\tilde{b}_{lh}(\mathbf{x}_k)$ can be further simplified and obtained as follows:

$$\widetilde{b}_{lh}(\mathbf{x}_k) = \frac{1}{A^{(k)}} \sum_{j=1}^{N_e^{(k)}} \frac{1}{3} A_e^j N_{l,h}^j$$

where A_e^j and $N_{l,h}$ are the area and derivative of shape function for the *jth* triangular element around the particle k, respectively.

Strain smoothing technique for SPFEM

The area of the smoothing cell $A^{(k)}$ is obtained as follows:

$$\mathcal{A}^{(k)} = \int_{\Omega^{(k)}} d\Omega = rac{1}{3} \sum_{j=1}^{N_e^{(k)}} \mathcal{A}_e^j$$

The smoothed strain-displacement operators \widetilde{B} at the *lth* node can be obtained as follows:

$$\widetilde{\boldsymbol{B}}_{L} = \begin{bmatrix} \widetilde{\boldsymbol{b}}_{lx} & \boldsymbol{0} \\ \boldsymbol{0} & \widetilde{\boldsymbol{b}}_{ly} \\ \widetilde{\boldsymbol{b}}_{ly} & \widetilde{\boldsymbol{b}}_{lx} \end{bmatrix}$$

□ Typical step of SPFEM:

Due to this special technique, Step 3 and Step 5 vanishes in the original PFEM.

 On the basis of cloud of particles, the Delaunay triangulation technique is used to build the FEM mesh.
 The alpha-shape approach is used to identify the entire problem domain.
 Map the state variables (strain, stress, etc.) from particles to Gauss points.
 Solve the governing equations via a standard incremental FEM approach.
 Map the state variables (strain, stress, etc.) from the Gauss points to the

particles

6. Modify the positions of particles and transfer all the field information of particles to form a new cloud of particles.

7. Go back to Step 1 and repeat until the problem-dependent stop condition.



与传统的PFEM相比,光滑粒子有限元法SPFEM有如下优点:

- □ 更像基于粒子的计算方法
- □ 避免了高斯点与节点之间的信息映射
- □ 使用低阶单元但没有体积锁定
- □ 对畸形单元不敏感







Implicit SPFEM

Similar to FEM and MPM, there are two types of SPFEM.

SPFEM SPFEM (Zhang, 2018) Explicit SPFEM (Yuan, 2019)



- Zhang W, Yuan W, Dai B. A smoothed particle finite element method for large-deformation problems in geomechanics, International Journal of Geomechanics, 2018, 18(4): 04018010
- 2. Yuan W H, Wang B, **Zhang W**, et al. Development of an explicit smoothed particle finite element method for geotechnical applications[J]. Computers and Geotechnics, 2019, 106: 42-51.

Implicit SPFEM

1. Governing equation

$$\rho \boldsymbol{a} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{b}$$

2. Weak form

$$\int_{V^{t}} \mathbf{D}_{ep} \cdot d\mathbf{\epsilon} \cdot \delta \mathbf{\epsilon} dV^{t} + \int_{V^{t}} (d\mathbf{\Omega} \cdot \mathbf{\sigma}^{t} + \mathbf{\sigma}^{t} \cdot d\mathbf{\Omega}^{T}) \cdot \delta \mathbf{\epsilon} dV^{t} + \int_{V^{t}} \mathbf{\sigma}^{t}$$
$$\cdot \delta \mathbf{\eta} dV^{t} = \int_{V^{t}} \mathbf{b}^{t+\Delta t} \cdot \delta \mathbf{u} dV^{t} + \int_{S^{t}} \mathbf{t}^{t+\Delta t} \cdot \delta \mathbf{u} dS^{t} - \int_{V^{t}} \mathbf{\sigma}^{t} \cdot \delta \mathbf{\epsilon} dV^{t}$$

3. discretization formulations

$$(\mathbf{K}_{\mathrm{ep}}+\mathbf{K}_{g})\mathbf{u}=\mathbf{F}^{\mathrm{ext}}$$

4. Newton-Raphson iteration

$$d\mathbf{u}_j = (\mathbf{K}_{j-1})^{-1} \mathbf{R}_{j-1}$$

 $\Delta \mathbf{u}_j = \Delta \mathbf{u}_{j-1} + d\mathbf{u}_j$
 $\mathbf{u}_j^{t+\Delta t} = \mathbf{u}^t + \Delta \mathbf{u}_j$

Implicit SPFEM

3. discretization formulations

$$(\mathbf{K}_{ep} + \mathbf{K}_{g})\mathbf{u} = \mathbf{F}^{ext} \qquad \mathbf{R}_{j-1} = \mathbf{F}^{ext} - \mathbf{F}^{int}$$

$$\mathbf{K}_{ep} = \int_{\Omega} \mathbf{B}_{L}^{T} \mathbf{D}_{ep} \mathbf{B}_{L} dV \qquad \mathbf{K}_{ep} = \sum_{k=1}^{N_{n}} \tilde{\mathbf{B}}_{L}^{(k)T} \mathbf{D}_{ep}^{(k)} \tilde{\mathbf{B}}_{L}^{(k)} A^{(k)}$$

$$\mathbf{K}_{g} = \int_{\Omega} \mathbf{B}_{L}^{T} \bar{\sigma} \mathbf{B}_{S} dV + \int_{\Omega} \mathbf{B}_{NL}^{T} \hat{\sigma} \mathbf{B}_{NL} dV \qquad \mathbf{K}_{g} = \sum_{k=1}^{N_{n}} \tilde{\mathbf{B}}_{L}^{(k)T} \bar{\sigma}_{k} \tilde{\mathbf{B}}_{S}^{(k)} A^{(k)} + \sum_{k=1}^{N_{n}} \tilde{\mathbf{B}}_{NL}^{(k)T} \hat{\sigma}_{k} \tilde{\mathbf{B}}_{NL}^{(k)} A^{(k)}$$

$$\mathbf{F}^{int} = \int_{\Omega} \mathbf{B}_{L}^{T} \sigma dV \qquad \mathbf{F}^{int} = \sum_{k=1}^{N_{n}} \tilde{\mathbf{B}}_{L}^{(k)T} \sigma_{k} A^{(k)}$$

$$\mathbf{Traditional FEM}$$
Nodal integration

Numerical procedure of iSPFEM

1. Read the particle information of the problem domain.

2. Loop over the load increment steps.

3. Generate the Delaunay triangles and identify the computational domain.

4. Construct the strain smoothing cells and compute the smoothed strain-displacement operators of all particles.

5. Solve the nonlinear equilibrium equations by the Newton-Raphson iteration to obtain particle incremental solutions of the current load incremental step.

6. Update the positions and field variables of particles.

7. End looping over the load increment steps.

8. Output results.



Explicit SPFEM

1. Governing equation

$$\rho \boldsymbol{a} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{b}$$

2. Weak form

$$\int_{V} \delta \boldsymbol{u} \cdot \rho \boldsymbol{a} dV = -\int_{V} \delta \boldsymbol{u} : \boldsymbol{\sigma} dV + \int_{S} \delta \boldsymbol{u} \cdot \boldsymbol{\tau}_{S} dS + \int_{V} \delta \boldsymbol{u} \cdot \rho \boldsymbol{b} dV$$

3. discretization formulations

$$Ma = F^{ext} - F^{int}$$

 $\boldsymbol{F}^{int} = \int_{\Omega} \tilde{\boldsymbol{B}}^T \boldsymbol{\sigma} d\Omega = \sum_{k=1}^T \sum_{i=1}^{N_k} \tilde{\boldsymbol{B}}_k^T \boldsymbol{\sigma}_k V_k \quad (\mathbf{\nabla} \boldsymbol{S}_k^T)$

Computational cycle of eSPFEM

Algorithm 1 Computational cycle of SPFEM

- 1. Generate Mesh using Delaunay triangulation and alpha shape technique
- 2. Get indices of elements related to nodes.
- 3. Calculate incremental time step and volumes of elements and nodes
- 4. Calculate smoothed strains of nodes
- 5. Calculate stresses of nodes through constitutive integration
- 6. Calculate internal force of nodes
- 7. Update positions and velocities of nodes by explicit time integration

3.
$$\Delta t_{cr} = \alpha \frac{l_{\min}}{c}$$

4. $\tilde{B}_k = \frac{1}{V_k} \sum_{i=1}^{N_k} \frac{1}{4} V^i B^i$
6. $F^{int} = \int_{\Omega} \tilde{B}^T \sigma d\Omega = \sum_{k=1}^T \sum_{i=1}^{N_k} \tilde{B}_k^T \sigma_k V_k$
7. $Ma = F^{ext} - F^{int}$

5. $\dot{\boldsymbol{\sigma}} = H(\dot{\boldsymbol{\epsilon}}, \kappa)$

The concise computational cycle greatly facilitate the GPU parallel computation

Extension to 3D and GPU parallel computation

1. Construct strain smoothing cells 2. Calculate the smoothed strain 2D case: $\boldsymbol{A}_k = \sum_{i=1}^{N_k} \frac{1}{3} A^i$ $ilde{oldsymbol{B}}_k = rac{1}{A_k} \sum_{i=1}^{N_k} rac{1}{3} A^i oldsymbol{B}^i$ 3D case: $\boldsymbol{V}_k = \sum_{i=1}^{N_k} \frac{1}{4} V^i$ $\tilde{\boldsymbol{B}}_k = \frac{1}{V_k} \sum_{i=1}^{N_k} \frac{1}{4} V^i \boldsymbol{B}^i$

The geometric data of smoothing cells are not required to be obtain explicitly.



Step 6. Calculate internal force of nodes



Smoothing strain matrices are too large! $(T \times N_k \times 18)$

- ✓ To save global memory, we reformulate the calculation of internal force.
- \checkmark Atomic operations are used to avoid racing conditions.

Parallelisation schemes for the eSPFEM



Zhang W, Zhong Z, Peng C, et al. GPU-accelerated smoothed particle finite element method for large deformation analysis in geomechanics. Computers & Geotechnics, 2021, doi: 10.1016/j.compgeo.2020.103856

PFEM for large deformation consolidation analysis



Yuan W, **Zhang W**^(*), Dai B, Application of the particle finite element method for large deformation consolidation analysis, Engineering Computations, 2019, 36(9): 3138-3163

PFEM with Abaqus



YUAN Wei-Hai, WANG Hao-Cheng, **ZHANG Wei**^(*), et al. Particle Finite Element Method implementation for large deformation analysis using Abaqus. Acta Geotechnica, 2021, doi: 10.1007/s11440-020-01124-2

Comments on SPFEM

D E Oñate, 2020, CMAME :

However, only very recently the use of nodal integration in a PFEM framework has been investigated and successfully applied to geotechnical problems [12,13]. In these works, the authors proposed the so-called Smoothed Particle Finite Element Method (SPFEM), inspired by the well-established Smoothed Finite Element Method (SFEM) [14–17].

■ E Oñate, 2020, ACME, A State of the Art Review of the Particle Finite Element Method (PFEM)



8.2 PFEM with Nodal Integration

Traditionally, the PFEM has been formulated for standard elemental integration, storing stresses and strains at the Gauss points. However, in PFEM with Gaussian integration, due to the continuous elimination of the elements done during the remeshing steps, it may be required to transfer the elemental information from the old mesh to the new one. This is avoided in fluid dynamics problems, where the measures of stresses and strains are computed from the scratch at each time step, but is mandatory for non-linear solid mechanics methods that require the storage of historical variables. Remapping procedures, besides having a certain computational cost, introduce interpolation errors into the numerical scheme and cause the smoothing of the historical variables (Sect. 2).

On the other hand, in nodal integration methods, stresses and material historical variables are computed and stored at the mesh nodes[99]. Consequently, a PFEM strategy with nodal integration does not require variable remapping procedures along the remeshing step.

This feature motivated recent research on the use of nodal integration in a PFEM framework[56, 124, 126]. The method, called by the authors Smoothed Particle Finite Element Method, took inspiration from the Smoothed Finite Element Method[135] and was successfully applied to 2D geomechanics problems with large deformations.





口 悬臂梁大变形



□ 悬臂梁大变形



口 悬臂梁大变形



Fig. 5. Deformed configurations and the horizontal stress σ_{xx} distribution (unit: megapascals): (a) magnitude of concentrated load is 1,500 kPa; (b) magnitude of concentrated load is 3,000 kPa





Fig. 6. Deformed configuration of the beam moment bending problem and the horizontal stress σ_{xx} distribution (unit: megapascals)

口 土体旁压试验

Cavity expansion in Tresca soil



Details in:

Zhang, W., Yuan, W.-H., Dai, B.-B., 2017. A smoothed particle finite element method for large-deformation problems in geomechanics. International Journal of Geomechanics. DOI: 10.1061/(ASCE)GM.1943-5622.0001079.

口 土体旁压试验

Cavity expansion in Tresca soil



Figure: Normalized internal pressure q/c_u versus normalized radial displacement b/a.

口 土体旁压试验

Cavity expansion in Tresca soil



Figure: Deformed configuration of the cavity expansion problem and the radial stress σ_r distribution (unit: kPa): (a) Normalized radial displacement b/a = 2; (b) Normalized radial displacement b/a = 3

口 软土地基上的条形基础



<u>隐式光滑粒子有限元法(iSPFEM)</u>

口 软土地基上的条形基础



Fig. 11. Mesh of the footing problem and the incremental deviatoric plastic strain invariant distribution: (a) normalized penetration, z/B = 0.5; (b) normalized penetration, z/B = 1.0

口 软土地基上的条形基础



Fig. 12. Normalized load-displacement curves for rigid footing on Tresca soil: (a) comparison with other numerical methods ($I_r = 33.4$); (b) comparison with PFEM at different I_r values

口 边坡大变形失稳

Failure of a homogeneous soil slope



Figure: Failure of a homogeneous soil slope

Details in:

Zhang, W., Yuan, W.-H., Dai, B.-B., 2017. A smoothed particle finite element method for large-deformation problems in geomechanics. International Journal of Geomechanics. DOI: 10.1061/(ASCE)GM.1943-5622.0001079.

口 边坡大变形失稳

Failure of a homogeneous soil slope



Figure: (a) Maximum displacement versus number of iterations; (b) Maximum displacement with different values of SRF.

口 边坡大变形失稳

Failure of a homogeneous soil slope



Figure: Configurations of the slope problem and the incremental deviatoric plastic strain invariant distribution: (a) SRF = 1.3; (b) SRF = 1.4; (c) SRF = 1.5; (d) SRF = 1.6

口 一维杆振动

Axial vibration of a continuum bar



口 软土地基上的条形基础

Rigid footing on Tresca soil



Figure: Geometry and mesh of the footing problem.

口 软土地基上的条形基础

Rigid footing on Tresca soil



Figure: Normalized load-displacement curves for rigid footing on Tresca soil: (a) Different particle densities; (b) Comparison with other numerical methods

口 软土地基上的条形基础

Rigid footing on Tresca soil



Figure: Contour of accumulated plastic strains (a) and shear stress (b) at a penetration depth of 2.5 m

口 砂柱垮塌

The collapse of two-dimensional sand columns



Figure: Initial setup (a) and final deposit (b) of Lube' s experiments.

口 砂柱垮塌

The collapse of two-dimensional sand columns



Figure: Normalized final height and width of granular columns as function of aspect ratio.



The collapse of two-dimensional sand columns



Figure: Final deposit profiles normalized to initial width for various aspect ratios.

口 铝棒垮塌







口 铝棒垮塌





口 长边坡渐进破坏



Fig. 15 Geometry and mesh for the strain-softening slope stability problem

口 长边坡渐进破坏



口 长边坡渐进破坏



口 铝棒垮塌



(Bui, 2008)



Mohr-Coulomb model is used













Fig. 14. Speedup of the double-precision GPU simulations over sequential GPU simulations.



Concept of the cohesion softening model



- ✓ With GPU acceleration, computation was completed in ~5 hours with ~250k nodes.
- ✓ Without GPU acceleration, computation was completed in ~13 hours with only ~ 38k nodes. (Zhang X, 2017)

水-力耦合粒子有限元法

20







surcharge load 50 kPa, p = 0

4R

-

30R

Fixed ur



Fixed u.

Abaqus粒子有限元法

6 D \mathbf{D} 口 管线-土相互作用 Free Surface (a) Ux = 0, Uy = 0.5DSoil: Smooth E = 1.0 MPa5 D v = 0.49 $c_a = 5.0 \ \mathrm{kPa}$ $\gamma = 1600 \text{ kg/m}^{3}$ (b) Ux = 0.5D, Uy = 0.5DRough 15 D Normalized horizontal displacements, u_x/D 0.0 0.5 1.0 1.5 (c) Ux = 1.0D, Uy = 0.5D0.05 Normalized resistance force, q/DC_u E0.02 Ð (d) Ux = 1.5D, Uy = 0.5D(e) Ux = 2.0D, Uy = 0.5D0 -0.0 0.5 Normalized vertical displacements, u_v/D Fig. 19 Normalized resistance force versus displacements

Smooth

2.0

Vertical resistance force -- Horizontal resistance force 5 D

基于SPFEM的CPT解析



基于SPFEM的CPT解析



基于SPFEM的CPT解析

In undrained penetration, the cone resistance qc is commonly related to the undrained shear strength by way of a relation of the form

$$q_{\rm c} = N_{\rm c} \mathbf{c}_{\rm u} + \boldsymbol{\sigma}_{\rm v0}$$

where σ_{v0} is the total overburden stress (e.g. Teh & Houlsby, 1991; Lu et al., 2004)



Cone factor with varying σ_{V0}

Cone factor with varying cu

Comparison against Existing Solutions





预制管桩贯入过程的模拟





Radial stress σ_r

Vertical stress σ_z

Installation of an open-ended pile



- 1. Zhang W, Yuan W, Dai B. Smoothed particle finite element method for large-deformation problems in geomechanics, International Journal of Geomechanics, 2018, 18(4): 04018010
- 2. Yuan W H, Wang B, **Zhang W**, et al. Development of an explicit smoothed particle finite element method for geotechnical applications. Computers and Geotechnics, 2019, 106: 42-51.
- 3. Yuan W, **Zhang W**^(*), Dai B, Application of the particle finite element method for large deformation consolidation analysis, Engineering Computations, 2019, 36(9): 3138-3163
- Yuan W, Liu K, Zhang W^(*), et al. Dynamic modeling of large deformation slope failure using smoothed particle finite element method, Landslides, 2020, doi: 10.1007/s10346-020-01375-w
- Zhang W, Zhong Z, Peng C, et al. GPU-accelerated smoothed particle finite element method for large deformation analysis in geomechanics. Computers & Geotechnics, doi: 10.1016/j.compgeo.2020.103856
- YUAN Wei-Hai, WANG Hao-Cheng, ZHANG Wei^(*), et al. Particle Finite Element Method implementation for large deformation analysis using Abaqus. Acta Geotechnica, 2021, doi: 10.1007/s11440-020-01124-2

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张巍,华南农业大学水利与土木工程学院,副教授,硕士生导师。 兼任中国土木工程学会土力学及岩土工程分会青年委员会委员、国 际土力学学会会员。2010年博士毕业于武汉大学水利水电工程专业, 2019年在维也纳自然资源与生命科学大学访学。主要研究方向为计 算土力学、根加固土理论。针对岩土工程中常见的滑坡、泥石流等 大变形问题,提出了岩土工程大变形数值模拟的光滑粒子有限元法 (SPFEM)基本框架,并将其推广至动力与耦合分析,又进一步实现 了三维GPU并行计算,为相关问题的研究提供了有效的数值工具。 主持广东省自然科学基金1项、广东省水利科技创新项目1项、广州 市荔湾区科技计划项目1项。共发表学术论文40余篇,其中SCI收录 21篇(第一/通讯作者16篇)。成果主要发表在岩土工程国际著名 期刊《Computers & Geotechnics》、《International Journal for numerical and analytical methods in geomechanics》、 《 Acta Geotechnica 》 、 《 International Journal of Geomechanics 》、《Canadian Geotechnical Journal》、 «Landslides» «Bulletin of Engineering Geology and the Environment》等。获全国优秀水利水电工程勘测设计奖金质奖1 项、广东省水利学会水利科学技术奖二等奖1项,授权发明专利2项, 获软件著作权1项。

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